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Optimizing Electric Grid Design Under Asymmetric Threat (II)

by

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March 2004

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13. ABSTRACT *(maximum 200 words)*

This research extends our earlier work to improve the security of electric power grids subject to disruptions caused by terrorist attacks. To identify critical system components (e.g., transmission lines, generators, transformers), we devise bilevel optimization models that identify maximally disruptive attack plans for terrorists, who are assumed to have limited offensive resources. A new model captures the dynamics of system operation as a network is repaired after an attack, and we adapt an earlier heuristic for that model's solution. We also develop a new, mixed-integer programming model (MIP) for the problem; a model that can be solved exactly using standard optimization software, at least in theory. Preliminary testing shows that optimal solutions are readily achieved for certain standard test problems, although not for the largest ones, which the heuristic seems to handle well. However, optimal solutions do provide a benchmark to measure the accuracy of the heuristic: The heuristic typically achieves optimality gaps of less than 10%, but occasionally the gap reaches 25%. Research will continue to refine the heuristic algorithm, the MIP formulation, and the algorithms to solve it. We also demonstrate progress made towards a graphical user interface that allows performing our interdiction analysis in a friendly environment.

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OPTIMIZING ELECTRIC GRID DESIGN UNDER ASYMMETRIC THREAT (II)

by

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Abstract

This research extends our earlier work to improve the security of electric power grids subject to disruptions caused by terrorist attacks. To identify critical system components (e.g., transmission lines, generators, transformers), we devise bilevel optimization models that identify maximally disruptive attack plans for terrorists, who are assumed to have limited offensive resources. A new model captures the dynamics of system operation as a network is repaired after an attack, and we adapt an earlier heuristic for that model's solution. We also develop a new, mixed-integer programming model (MIP) for the problem; a model that can be solved exactly using standard optimization software, at least in theory. Preliminary testing shows that optimal solutions are readily achieved for certain standard test problems, although not for the largest ones, which the heuristic seems to handle well. However, optimal solutions do provide a benchmark to measure the accuracy of the heuristic: The heuristic typically achieves optimality gaps of less than 10%, but occasionally the gap reaches 25%. Research will continue to refine the heuristic algorithm, the MIP formulation, and the algorithms to solve it. We also demonstrate progress made towards a graphical user interface that allows performing our interdiction analysis in a friendly environment.

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1. INTRODUCTION

This report documents the continuing research project entitled "Homeland Security Research And Technology Proposal (Optimizing Electric Grid Design Under Asymmetric Threat)," which is sponsored by the U.S. Department of Justice, Office of Justice Programs and Office of Domestic Preparedness.

This research extends our previous effort aimed at developing new optimization models and methods for planning expansion and enhancements of electric power grids that improve robustness to potential disruptions caused by natural disasters, sabotage and, especially, terrorist attacks. The research reported here shows the progress made in different areas comprising modeling, algorithms and their implementation, testing, and user interfaces.

The document is organized as follows: Section 2 presents an overview of project accomplishments to date. Sections 3 through 7 describe this year's activities in detail. In particular, Section 3 describes our model of post-attack system restoration over time, and an associated solution procedure. Section 4 focuses on a new model representation as a standard mixed-integer program. Computational results for these models are presented in Section 5, including comparisons with earlier results. Section 6 presents an overview of our new "VEGA" decision-support system, which includes database and graphical user-interface tools, along with an optimization module. Other activities are summarized in Section 7. Section 8 presents an overview of the work intended for year 2004. We conclude in Section 9 with the list of criteria used to assess the value of our project to Homeland Security.

2. OBJECTIVE AND SUMMARY OF WORK COMPLETED

Our project develops new mathematical models and optimization methods for robust planning of electrical power grids, focusing on security and reliability, with special emphasis on potential disruptions caused by terrorist attacks. We refer to our previous proposal [Salmeron and Wood, 2002-I] and references therein for detailed background on the problem of electric power-grid vulnerability.

A statement from the Committee on Science and Technology for Countering Terrorism [2002] succinctly states the motivation for our work: "The nation's electric power systems must clearly be made more resilient to terrorist attack." This motivation has been further strengthened

by the blackout, on 14 August 2003, in the Northeast U.S. electric power grid [U.S.-Canada Power System Outage Task Force, 2003]. Although the blackout was not instigated by terrorists, its cause appears to have been multiple failures of infrastructure elements in the transmission system, and our current research addresses precisely such situations.

We also refer to our previous report [Salmeron, Wood, and Baldick, 2003-I], where we establish the mathematical foundations for the models and algorithms that we have enhanced in the research reported here.

The following milestones have been achieved as the result of previous and current research (see proposals by Salmeron and Wood [2002-I], [2002-II]). (Underlined items identify the most recent contributions.)

(1) Formulation of mathematical models that represent the problem of optimally interdicting an electrical power grid. A newer formulation includes system restoration over time.

(2) Development of heuristics that identify highly disruptive attack plans to a specific electric power grid given limited interdiction resources. Newer heuristics keep pace with the developing models and have been adapted to incorporate system restoration over time. This algorithm has been implemented using the General Algebraic Modeling Language software [GAMS, 1996].

(3) Incorporation of different measures of effectiveness, any of which can be optimized:
- Short-term power disruption (MW);
- Short-term cost ($/MW) over all consumer sectors;
- Long-term energy disruption (MWh), including system restoration over time; and
- Long-term cost ($) over all consumer sectors, including system restoration over time.

(4) Development of techniques to convert models in (1) into standard mixed-integer programs that can be solved exactly. This means that, not only we can determine "good" attack plans, as our heuristic approach (2) does, but we can prove that these plans are optimal. In turn, this provides us with a precise measure of vulnerability. We have implemented and solved the converted models using GAMS [1996].

(5) Solution to cases with up to 100 electrical buses (drawn from the IEEE Reliability Test Data [1999-I], [1999-II]).

(6) Presentations in the Homeland Security Leadership Development (HSLD) seminars:

- "Electric Power Grids Vulnerability," CS4920, Naval Postgraduate School (3 December 2002).
- "Vulnerability of Electric Power Grids," CS3660, Naval Postgraduate School (18 June 2003).

(7) Reports:

- First-year report [Salmeron, Wood, and Baldick, 2003-I].
- Research Paper [Salmeron, Wood, and Baldick, 2003-II], accepted for publication in *IEEE Transactions on Power Systems*.

(8) Thesis students involvement:

- Major Dimitrios Stathakos, Greek Army. "An Enhanced Graphical User Interface for Analyzing the Vulnerability of Electrical Power Systems to Terrorist Attacks," [Stathakos, 2003], graduated in December 2003.
- LCDR Rogelio Alvarez, USN. "Interdicting Electrical Power Grids," graduation expected in March 2004.

(9) Graphical User Interface (GUI) Development: We realize the importance of enabling access to this type of analysis to a number of potential users, who are not necessarily familiar with the optimization arena. To bridge this gap, we have initiated the design and implementation of the "Vulnerability of Electrical Power Grids Analyzer" (VEGA) system. VEGA is an integrated tool, comprising a graphical user interface (GUI), a supporting database (DB) and the aforementioned optimization tools. A preliminary Web page has been set up for this project [VEGA, 2003]. VEGA 1.0 is the first prototype of this system.

We next describe the items (1)-(9) above in more detail.

3. MODELING RESTORATION AND ALGORITHMIC IMPLEMENTATION

3.1 Previous Interdiction Model without Restoration

The mathematical model we presented in our previous report [Salmeron, Wood, and Baldick, 2003-I] attempts to maximize immediate electric power shedding by (optimally) selecting a set of interdictions given limited resources. We refer to that report for a full description of its formulation, which can be shortly stated as the following Max-min (Mm) problem:

$$
\begin{aligned}
\text{(Mm)} \qquad & \max_{\boldsymbol{\delta}\in\Delta} \min_{\mathbf{p}} \mathbf{c}'\mathbf{p} \\
& \quad \text{s.t. } \mathbf{g}(\mathbf{p},\boldsymbol{\delta}) \le \mathbf{b} \\
& \qquad\qquad \mathbf{p} \ge \mathbf{0}
\end{aligned}
$$

Recall that, in this model, an interdiction plan is represented by the binary vector $\boldsymbol{\delta}$, whose k-th entry δ_k is 1 if component k of the system is attacked and is 0 otherwise. For a given plan, the inner problem (called **DC-OPF**) is an optimal power-flow model [Wood and Wollenberg, 1996, p. 514] that minimizes generation costs plus the penalty associated with unmet demand, together denoted by $\mathbf{c}'\mathbf{p}$. Here, $\mathbf{p}$ represents power flows, generation outputs, phase angles, and "unmet demand," i.e., the amount of load shed; $\mathbf{c}$ represents linearized generation costs and the costs of unmet demand. The outer maximization chooses the most disruptive, resource-constrained interdiction plan $\boldsymbol{\delta}\in\Delta$, where Δ is a discrete set representing attacks that a terrorist group might be able to carry out. In this model, $\mathbf{g}$ corresponds to a set of functions that are nonlinear in $(\mathbf{p},\boldsymbol{\delta})$.

The inner problem involves a simplified optimal power-flow model, with constraint functions $\mathbf{g}(\mathbf{p},\boldsymbol{\delta})$ that are, however, linear in $\mathbf{p}$ for a fixed $\boldsymbol{\delta}=\hat{\boldsymbol{\delta}}$.

3.2 Interdiction Model with Restoration

<u>Overview</u>

Model (Mm) provides only a rough estimate of energy shedding and thus the true cost to society of an attack on a power grid. This is because (Mm) is based on the system capability after the initial return to service of non-damaged equipment following an attack, disregarding medium- and long-term effects. The only case in which this is not important is when the outage duration of

all interdictable components in the system is the same, which seems unlikely (e.g., interdicted substations will, in general, take much longer to repair than interdicted lines).

We have extended model (Mm) to handle the cost and timing of repairs, which allows us to obtain interdiction plans seeking to maximize total cost as the system is restored over time. Essentially, this model measures "total weighted energy," where weights represent costs of lost energy to various customer sectors and possibly other factors.

This is accomplished by using interdiction constructs to couple instances of DC-OPF, one for each system state that represents a stage or "time period" of system repair. In outline, the model is:

$$
\begin{aligned}
(\text{Mm}') \qquad & \max_{\boldsymbol{\delta}\in\Delta}\ \min_{\mathbf{p}} \sum_{t\in T} D_t \mathbf{c}'\mathbf{p}_t \\
& \text{s.t. } \mathbf{g}_t(\mathbf{p}_t,\boldsymbol{\delta}) \le \mathbf{b} \quad \forall t\in T \\
& \qquad \mathbf{p}_t \ge \mathbf{0} \quad \forall t\in T.
\end{aligned}
$$

Model (Mm′) extends (Mm) to incorporate the hourly cost of power flow, $\mathbf{c}'\mathbf{p}_t$, in each time period *t*, multiplied by the period's duration in hours D_t. Figure 1 shows the difference between potential solutions provided by (Mm) and (Mm′).

The model could be further extended to incorporate "sub-time periods" through load duration curves, but we have not yet explored this possibility; all loads are held constant over time.

Following the notation and conventions in our previous report (see Appendix A), we next describe the full model that incorporates system restoration. We first need to introduce some additional notation:

$T =$ set of periods, for $t \in T$

$\xi = L^* \cup G^* \cup B^* \cup S^*$, set of all (directly) interdictable elements

$Dur(e) =$ Duration (hours) of outage for element $e \in \xi$, if attacked

$D_t =$ Duration (hours) of time period t, for $t \in T$

$$\beta_{t,e} = \begin{cases} 1, & \text{if component } e \text{ remains unrepaired in time period } t \text{ after being attacked} \\ 0, & \text{if component } e \text{ is repaired before time period } t \text{ after being attacked} \end{cases},$$

for $t \in T$, $e \in \xi$.

Remark: In the above notation $\beta_{t,l}^{\text{Line}}$, $\beta_{t,i}^{\text{Bus}}$, $\beta_{t,g}^{\text{Gen}}$, and $\beta_{t,s}^{\text{Sub}}$ denote $\beta_{t,e}$ when *e*=*l* is a line, or *e*=*i* is a bus, or *e*=*g* is a generator, or *e*=*s* is a substation, respectively.

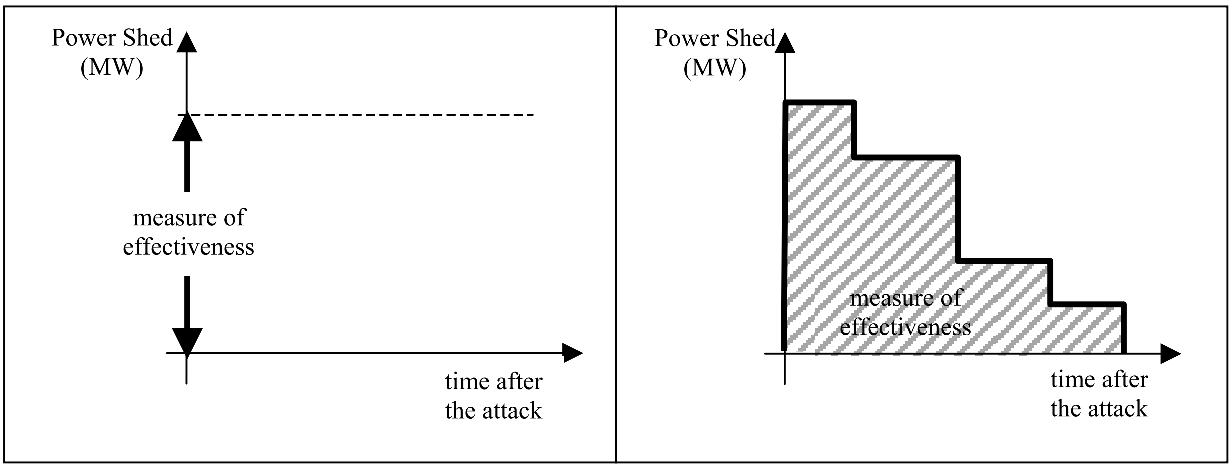

Figure 1. The model without restoration provides the optimal interdiction plan according to instantaneous power shed, after initial return to service of non-damaged equipment (left). The model with restoration over time, accounts for energy disruption (right).

The following algorithm constructs the set of time periods, *T*, based on the different outage durations for all interdictable elements. In the course of the algorithm, D_t and $\eta_{t,e}$ are also constructed:

Algorithm "Construct Time Periods":

Initialization: $\tilde{\xi} \leftarrow \xi,\ T \leftarrow \varnothing,\ t \leftarrow 0,\ m_0 \leftarrow 0;$

While $\tilde{\xi} \neq \varnothing$:

$$t \leftarrow t+1$$

$$T \leftarrow T \cup \{t\}$$

$$\beta_{t,e} \leftarrow \begin{cases} 1, \text{ if } e \in \tilde{\xi} \\ 0, \text{ otherwise} \end{cases}$$

$$m_t \leftarrow \min\left\{ Dur(e) \middle| e \in \tilde{\xi} \right\}$$

$$\xi_t \leftarrow \left\{ e \middle| e \in \tilde{\xi} \wedge Dur(e) = m_t \right\}$$

$$D_t \leftarrow m_t - m_{t-1}$$

$$\tilde{\xi} \leftarrow \tilde{\xi} \setminus \xi_t$$

End While

Due to different outage duration for the system components, we need to establish which components might be out of service during each time period. For example, if a line l can be interdicted, but it is not connected to an interdictable bus, then the line is guaranteed to be in service after $Dur(l)$ hours, independent of whether it is attacked or not. At this point, the following definitions are needed:

L_t^{**} = Set of lines l that could be out of service in period t following a direct or indirect interdiction.

G_t^{**} = Set of generators g that could be out of service in period t following a direct or indirect interdiction.

$\beta_{t,e}$ helps us define these sets precisely. In particular:

$$l \in L_t^{**} \text{ if either: } \begin{cases} \beta_{t,l}^{\text{Line}} = 1, \text{ or} \\ \beta_{t,i}^{\text{Bus}} = 1 \text{ for some } i \mid l \in L_i^{Bus}, \text{ or} \\ \beta_{t,s}^{\text{Sub}} = 1 \text{ for some } s \mid l \in L_s^{Sub}, \text{ or} \\ \beta_{t,ll}^{\text{Line}} = 1 \text{ for some } ll \mid ll \in L_l^{Par} \end{cases}$$

$$g \in G_t^{**} \text{ if either: } \begin{cases} \beta_{t,g}^{\text{Gen}} = 1, \text{ or} \\ \beta_{t,i(g)}^{\text{Bus}} = 1, \text{ or} \\ \beta_{t,s(i(g))}^{\text{Sub}} = 1 \end{cases}$$

Additional definitions are:

$$\lambda_l^L = \begin{cases} 1, \text{ if } l \in L^* \\ 0, \text{ otherwise} \end{cases}, \quad \lambda_{t,l}^L = \begin{cases} 1, \text{ if } l \in L_t^{**} \\ 0, \text{ otherwise} \end{cases}$$

$$\lambda_g^G = \begin{cases} 1, \text{ if } g \in G^* \\ 0, \text{ otherwise} \end{cases}, \quad \lambda_{t,g}^G = \begin{cases} 1, \text{ if } g \in G_t^{**} \\ 0, \text{ otherwise} \end{cases}$$

$$\lambda_i^I = \begin{cases} 1, \text{ if } i \in I^* \\ 0, \text{ otherwise} \end{cases}$$

$$\lambda_s^S = \begin{cases} 1, \text{ if } s \in S^* \\ 0, \text{ otherwise} \end{cases}$$

(Hereafter, equations in boxes represent final constraints in our models.)

The objective function of the interdiction problem with restoration (I-DC-OPF-R) becomes:

$$\boxed{\max_{\boldsymbol{\delta} \in \Delta} \min_{\substack{(P_t^{Gen}, P_t^{Line}, \\ S_t, \theta_t) \in \Gamma}} \sum_{t \in T} D_t \cdot \left\{ \sum_g h_g P_{t,g}^{Gen} + \sum_i \sum_c f_{i,c} S_{t,i,c} \right\}} \quad \text{(I-R.0)}$$

where, as in the case without time periods, we attempt to minimize power generation cost plus load shedding cost, but this time these terms are specified by time period and weighed by its duration, in order to account for energy cost.

$\boldsymbol{\delta} \in \Delta$ still represents the resource-constrained interdictions that a terrorist group might be able to carry out, and is the same as in the case without time periods. Or, explicitly:

$$\boxed{\sum_{g \in G^*} M_g^{Gen} \delta_g^{Gen} + \sum_{l \in L^*} M_l^{Line} \delta_l^{Line} + \sum_{i \in I^*} M_i^{Bus} \delta_i^{Bus} + \sum_{s \in S^*} M_s^{Sub} \delta_s^{Sub} \le M} \quad \text{(I-R.1)}$$

$$\boxed{\text{All } \delta\text{-variables are binary}} \quad \text{(I-R.2)}$$

$(P_t^{Gen}, P_t^{Line}, \theta_t, S_t) \in \Gamma$ represents the time-dependent decision variables of the inner DC-OPF-R (DC-OPF model with restoration). These constraints (and the associated duals, denoted as π with appropriate sub- and super-indices) are explicitly stated as:

- Admittance equation for line *l*, subject to possibly being out of service during period *t*:

$$\boxed{\begin{aligned} P_{t,l}^{Line} = B_l(\theta_{t,o(l)} - \theta_{t,d(l)})(1-\lambda_l^{Line}\beta_{t,l}^{Line}\delta_l^{Line}) \prod_{i \in I^* \mid l \in L_i^{Bus}} (1-\beta_{t,i}^{Bus}\delta_i^{Bus}) \cdot \\ \prod_{s \in S^* \mid l \in L_s^{Sub}} (1-\beta_{t,s}^{Sub}\delta_s^{Sub}) \prod_{ll \in L^* \mid ll \in L_l^{Line}} (1-\beta_{t,ll}^{Line}\delta_{ll}^{Line}) \quad \forall l, \forall t \qquad (\pi_{t,l}^{A^-}) \end{aligned}} \tag{IDC-R.1}$$

- Balance equation for the bus *i* in period *t*:

$$\boxed{\sum_{g \in G_i} P_{t,g}^{Gen} - \sum_{l \mid o(l)=i} P_{t,l}^{Line} + \sum_{l \mid d(l)=i} P_{t,l}^{Line} + \sum_c S_{t,i,c} = \sum_c d_{i,c} \quad \forall i,t \qquad (\pi_{t,i}^{Bal})} \tag{IDC-R.2}$$

- Line *l* capacity, subject to possibly being out of service during period *t*:

$$\boxed{\begin{array}{lll}
P_{t,l}^{Line} \le \overline{P}_l^{Line} & \forall t, l \notin L_t^{**} & (\pi_{t,l}^{L_0^-}) \\
P_{t,l}^{Line} \le \overline{P}_l^{Line}(1-\delta_l^{Line}) & \forall t, l \in L^*,\ \beta_{t,l}^{Line}=1 & (\pi_{t,l}^{LCap^-}) \\
P_{t,l}^{Line} \le \overline{P}_l^{Line}(1-\delta_i^{Bus}) & \forall t,l,i \mid i \in I^*,\ l \in L_i^{Bus}, \beta_{t,i}^{Bus}=1 & (\pi_{t,l,i}^{LB^-}) \\
P_{t,l}^{Line} \le \overline{P}_l^{Line}(1-\delta_s^{Sub}) & \forall t,l,s \mid s \in S^*, l \in L_s, \beta_{t,s}^{Sub}=1 & (\pi_{t,l,s}^{LS^-}) \\
P_{t,l}^{Line} \le \overline{P}_l^{Line}(1-\delta_{ll}^{Line}) & \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, \beta_{t,ll}^{Line}=1 & (\pi_{t,l,ll}^{LL^-}) \\
P_{t,l}^{Line} \ge -\overline{P}_l^{Line} & \forall t, l \notin L_t^{**} & (\pi_{t,l}^{L_0^+}) \\
P_{t,l}^{Line} \ge -\overline{P}_l^{Line}(1-\delta_l^{Line}) & \forall t, l \in L^*,\ \beta_{t,l}^{Line}=1 & (\pi_{t,l}^{LCap^+}) \\
P_{t,l}^{Line} \ge -\overline{P}_l^{Line}(1-\delta_i^{Bus}) & \forall t,l,i \mid i \in I^*,\ l \in L_i^{Bus}, \beta_{t,i}^{Bus}=1 & (\pi_{t,l,i}^{LB^+}) \\
P_{t,l}^{Line} \ge -\overline{P}_l^{Line}(1-\delta_s^{Sub}) & \forall t,l,s \mid s \in S^*, l \in L_s, \beta_{t,s}^{Sub}=1 & (\pi_{t,l,s}^{LS^+}) \\
P_{t,l}^{Line} \ge -\overline{P}_l^{Line}(1-\delta_{ll}^{Line}) & \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, \beta_{t,ll}^{Line}=1 & (\pi_{t,l,ll}^{LL^+})
\end{array}} \tag{IDCR.3}$$

It is worth noting that, for the each line l, we need five constraints to specify its maximum capacity in period t, and another five constraints for the minimum capacity. For example, consider the case where a line l can be interdicted, but it is not connected to an interdictable bus or substation. Then, $l \in L_t^{**}$ for some of the first periods t (e.g., t=1,2,3), but since the line is guaranteed to be back in service after $Dur(l)$ hours, $l \notin L_t^{**}$ for the rest of the periods (e.g., t=4,5,6). Assuming also that the line has no other lines in parallel, the constraints to be used will be:

$$P_{t,l}^{Line} \leq \overline{P}_l^{Line} \qquad t = 4,5,6$$

$$P_{t,l}^{Line} \leq \overline{P}_l^{Line}(1-\delta_l^{Line}) \qquad t = 1,2,3$$

On the other hand, if the line is connected to an interdictable bus, and the bus outage covers, for example, periods t=1,...,5, the constraints would be:

$$P_{t,l}^{Line} \leq \overline{P}_l^{Line} \qquad t = 6$$

$$P_{t,l}^{Line} \leq \overline{P}_l^{Line}(1-\delta_l^{Line}) \qquad t = 1,2,3$$

$$P_{t,l}^{Line} \leq \overline{P}_l^{Line}(1-\delta_i^{Bus}) \qquad t = 1,2,3,4,5$$

- Generator g maximum output, subject to possibly being out of service during period t:

$$\boxed{\begin{array}{lll} P_{t,g}^{Gen} \leq \overline{P}_g^{Gen} & \forall t, g \notin G_t^{**} & (\pi_{t,g}^{G_0}) \\ P_{t,g}^{Gen} \leq \overline{P}_g^{Gen}(1-\delta_g^{Gen}) & \forall t, g \mid g \in G^*, \beta_{t,g}^{Gen}=1 & (\pi_{t,g}^{G}) \\ P_{t,g}^{Gen} \leq \overline{P}_g^{Gen}(1-\delta_{i(g)}^{Bus}) & \forall t, g \mid i(g) \in I^*, \beta_{t,i(g)}^{Bus}=1 & (\pi_{t,g}^{GB}) \\ P_{t,g}^{Gen} \leq \overline{P}_g^{Gen}(1-\delta_{s(i(g))}^{Sub}) & \forall t, g \mid s(i(g)) \in S^*, \beta_{t,s(g)}^{Sub}=1 & (\pi_{t,g}^{GS}) \end{array}} \qquad \text{(IDC-R.4)}$$

- Demand shedding at bus i for customer c:

$$\boxed{S_{t,i,c} \leq d_{i,c} \qquad \forall i,c \qquad (\pi_{t,i,c}^{Load})} \qquad \text{(IDC-R.5)}$$

- Variable sign:

$$\boxed{\begin{array}{ll} P_{t,g}^{Gen} \geq 0 & \forall t,g \\ P_{t,l}^{Line} \text{ unrestricted} & \forall t,l \\ S_{t,i,c} \geq 0 & \forall t,i,c \\ \theta_{t,i} \text{ unrestricted} & \forall t,i \end{array}} \qquad \text{(IDC-R.6)}$$

In summary, our interdiction model with restoration, (I-R), becomes:

$$\text{(I-R):} \quad \max_{\boldsymbol{\delta}} \min_{P_t^{Gen}, P_t^{Line}, S_t, \theta_t} \quad \text{(I-R.0)}$$

subject to:

(I-R.1), (I-R.2) and

(IDC-R.1) to (IDC-R.6)

3.3 Heuristic Algorithm for the Interdiction Problem with Restoration

The algorithm that we use to solve (I-R), i.e., a problem without restoration, is schematically depicted in Figure 2.

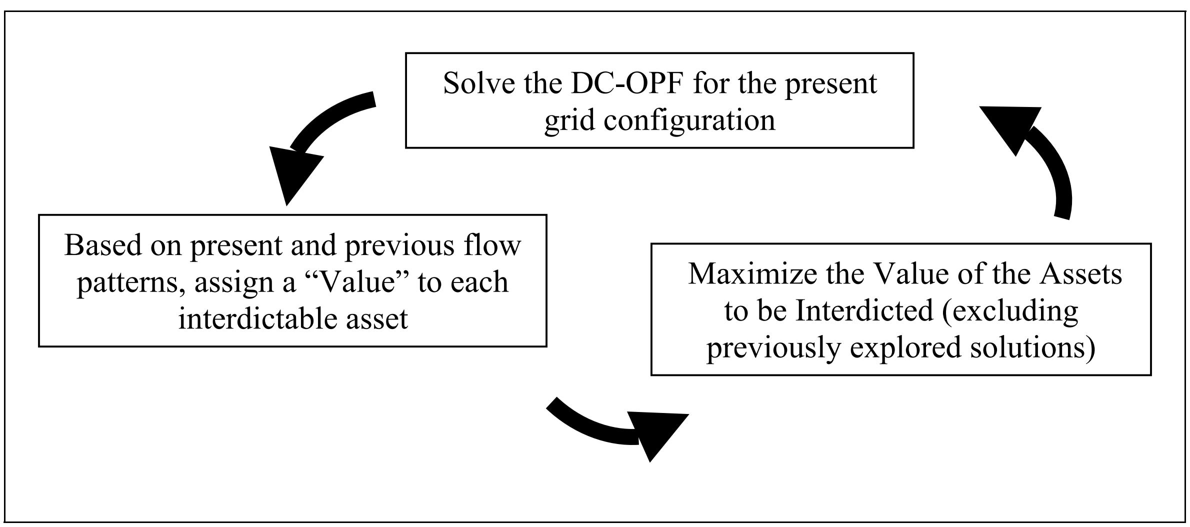

Figure 2: Interdiction algorithm framework (without restoration).

This algorithm first solves DC-OPF assuming no attacks. The power-flow pattern is used to assign relative values to all the components of the power grid. These values are used to maximize

the estimated value of the assets to be interdicted, while ensuring that the resources required for the interdiction plan are not exceeded. With this interdiction plan, we modify the right-hand side of DC-OPF model (disregarding restoration) and obtain its solution. In this case, we expect part of the load to be shed. The process continues by finding alternative sets of valuable assets to interdict that have not been identified at earlier iterations, and by evaluating load shedding for each of these interdiction plans. More details on the algorithm can be found in Salmeron et al. [2003-II].

We can adapt this algorithm to the model with restoration by solving the new DC-OPF problem in the upper box in Figure 2, assuming that all δ-variables have been fixed (say $\boldsymbol{\delta} = \hat{\boldsymbol{\delta}}$, satisfying (I-R.1) and (I-R.2)). That problem, denoted DC-OPF-R($\hat{\boldsymbol{\delta}}$), becomes:

$$\text{DC-OPF-R}(\hat{\boldsymbol{\delta}}): \qquad \min_{P_t^{Gen}, P_t^{Line}, S_t, \theta_t} \text{(I-R.0)}$$

$$\text{subject to: (IDC-R.1) to (IDC-R.6)}$$

DC-OPF-R($\hat{\boldsymbol{\delta}}$) (called "sub-problem" in the above algorithm, for a specific interdiction plan $\hat{\boldsymbol{\delta}}$), provides the joint power flow patterns for a number of system "stages": one for each restoration period. Notice that DC-OPF-R($\hat{\boldsymbol{\delta}}$) decomposes into $|T|$ sub-sub-problems, each of which consists of an instance of DC-OPF with some subset of system components being "out of service." Outaged components are determined by $\hat{\boldsymbol{\delta}}$ (interdictions) and by $\beta_{t,e}$, which informs DC-OPF about the status of interdicted components in period *t*.

In addition to this modification, our heuristic algorithm also redefines the concept of "value," which is used to determine which grid components appear more attractive for interdiction in each iteration. We maintain the same concept of value (denoted as a vector **V**) as in our previous work (again, see details in Salmeron, Wood, and Baldick [2003-I]), but noticing that these values must be multiplied by $Dur(e)$ (for a generic component, *e*) in order to account for energy-based values. Therefore, assuming the definitions of value from previous work, the new definition of value for every generic component *e* is:

$$\mathrm{V}^{\text{Restoration}}(e) = Dur(e) \times \mathrm{V}^{\text{No-Restoration}}(e)$$

With these values, the "master problem" can find potentially good interdiction plans $\hat{\boldsymbol{\delta}}$ that have not been explored yet. The master problem (MP-R) at a specific iteration τ is:

$$\text{MP-R}(\mathbf{V}^{\tau}, \hat{\Delta}^{\tau}): \qquad \max_{\boldsymbol{\delta}} \ \mathbf{V}^{\tau} \cdot \delta$$

subject to:

Eqs. (I-R.1) - (I-R.2) replacing $\boldsymbol{\delta}$ with $\boldsymbol{\delta}^{\tau}$, and:

$$\boxed{\begin{array}{ll} \delta_g^{Gen} + \delta_i^{Bus} \le 1 & \forall g \in \mathbf{G}_i^*, i \in \mathbf{I}^* \\ \delta_l^{Line} + \delta_i^{Bus} \le 1 & \forall l \in \mathbf{L}_i \cap \mathbf{L}^*, i \in \mathbf{I}^* \\ \delta_l^{Line} + \delta_{l'}^{Line} \le 1 & \forall l' \in \mathbf{L}_l^{Par} \cap \mathbf{L}^*, l \in \mathbf{L}^* \\ \delta_i^{Bus} + \delta_s^{Sub} \le 1 & \forall i \in \mathbf{I}_s \cap \mathbf{I}^*, s \in \mathbf{S}^* \\ \delta_l^{Line} + \delta_s^{Sub} \le 1 & \forall l \in \mathbf{L}_s \cap \mathbf{L}^*, s \in \mathbf{S}^* \end{array}} \qquad \text{(MP-R.1)}$$

$$\boxed{\begin{array}{l} \displaystyle\sum_{\substack{g \in \mathbf{G}^* | \\ \hat{\delta}_g^{Gen,\tau'}=1}} (\hat{\delta}_g^{Gen,\tau'} - \delta_g^{Gen}) + \sum_{\substack{l \in \mathbf{L}^* | \\ \hat{\delta}_l^{Line,\tau'}=1}} (\hat{\delta}_l^{Line,\tau'} - \delta_l^{Line}) + \\ \quad \displaystyle\sum_{\substack{i \in \mathbf{I}^* | \\ \hat{\delta}_i^{Bus,\tau'}=1}} (\hat{\delta}_i^{Bus,\tau'} - \delta_i^{Bus}) + \sum_{\substack{s \in \mathbf{S}^* | \\ \hat{\delta}_s^{Sub,\tau'}=1}} (\hat{\delta}_s^{Sub,\tau'} - \delta_s^{Sub}) \ge 1 \quad \forall \tau' \le \tau. \end{array}} \qquad \text{(MP-R.2)}$$

where $\hat{\Delta}^{\tau}$ is a set that contains the information on all previously-generated interdiction plans. The first block of constraints (MP-R.1) are valid inequalities to ensure that a system component is not interdicted if it has been indirectly interdicted by an element to which it is connected. (Remark: The valid inequalities (MP-R.2) also need to be adjusted when considering system restoration because they were based on arguments that ignored repairs over time.) The second block (MP-R.1) are super-valid inequalities that account for previously generated solutions in the algorithm, in order to always examine alternative solutions.

Figure 3 sketches a framework for the new algorithm.

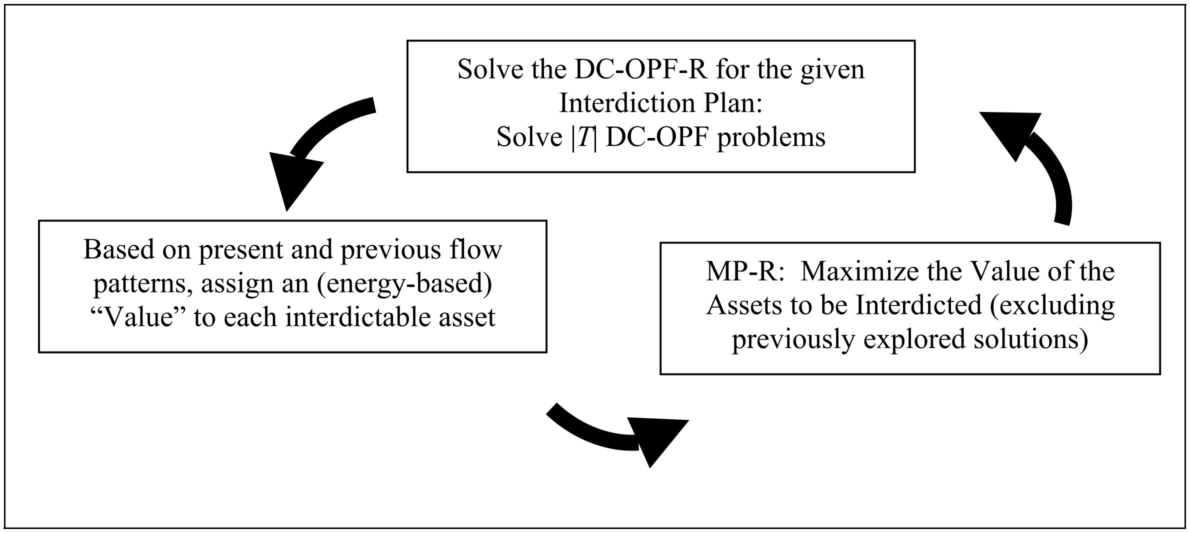

Figure 3: Interdiction algorithm framework (with restoration).

A more detailed version of this algorithm (I-ALG-R) follows:

I-ALG-R

Input: Grid data; Interdiction data; $\tau^{\max}$ (iteration limit).

Output: $\hat{\boldsymbol{\delta}}^*$ is a feasible interdiction plan causing a disruption with cost $\boldsymbol{\gamma}^*$. If the algorithm exits because $\mathrm{MP}(\mathbf{V}^\tau, \hat{\Delta}^\tau)$ is infeasible, then all feasible plans have been enumerated, and $\hat{\boldsymbol{\delta}}^*$ is therefore optimal.

Initialization:

- $\hat{\boldsymbol{\delta}}^1 \leftarrow (\hat{\boldsymbol{\delta}}^{Gen,1}, \hat{\boldsymbol{\delta}}^{Line,1}, \hat{\boldsymbol{\delta}}^{Bus,1}, \hat{\boldsymbol{\delta}}^{Sub,1}) \leftarrow (\mathbf{0},\mathbf{0},\mathbf{0},\mathbf{0})$ (initial attack plan).
- $\hat{\boldsymbol{\delta}}^* \leftarrow \hat{\boldsymbol{\delta}}^1$ (best plan so far) and $\hat{\Delta}^1 \leftarrow \{\hat{\boldsymbol{\delta}}^1\}$.
- $\gamma^* \leftarrow 0$ (cost of the best plan so far).
- $\tau \leftarrow 1$.

Subproblem:

- Solve $\text{DC-OPF-R}(\hat{\boldsymbol{\delta}}^\tau)$ for objective value $\gamma(\hat{\boldsymbol{\delta}}^\tau)$ and solution $\hat{\mathbf{P}}^\tau = (\hat{\mathbf{P}}^{Line,\tau}, \hat{\mathbf{P}}^{Gen,\tau}, \hat{\mathbf{S}}^\tau, \hat{\boldsymbol{\theta}}^\tau)$.
- If $\gamma(\hat{\boldsymbol{\delta}}^\tau) > \gamma^*$, then $\gamma^* \leftarrow \gamma(\hat{\boldsymbol{\delta}}^\tau)$, and $\hat{\boldsymbol{\delta}}^* \leftarrow \hat{\boldsymbol{\delta}}^\tau$.
- If $\tau = \tau^{\max}$, then Print ($\hat{\boldsymbol{\delta}}^*$, γ^*) and halt.

Master Problem:

- Compute estimated values:

$$\mathbf{V}^{\tau} \equiv \left(\mathbf{V}^{Gen,\tau}, \mathbf{V}^{Line,\tau}, \mathbf{V}^{Bus,\tau}, \mathbf{V}^{Sub,\tau}\right) \leftarrow \frac{1}{\tau}\sum_{\tau'=1}^{\tau}\left(\mathbf{V}^{Gen,\tau'}, \mathbf{V}^{Line,\tau'}, \mathbf{V}^{Bus,\tau'}, \mathbf{V}^{Sub,\tau'}\right).$$

- Solve $\text{MP-R}(\mathbf{V}^{\tau}, \hat{\Delta}^{\tau})$ for $\hat{\boldsymbol{\delta}}^{t+1}$.
- If $\text{MP-R}(\mathbf{V}^{\tau}, \hat{\Delta}^{\tau})$ is infeasible, then Print ($\hat{\boldsymbol{\delta}}^{*}$, γ^{*}) and halt.
- $\hat{\Delta}^{\tau+1} \leftarrow \hat{\Delta}^{\tau} \cup \{\hat{\boldsymbol{\delta}}^{\tau+1}\}$.
- $\tau \leftarrow \tau + 1$.
- Return to Subproblem.

4. MIXED-INTEGER REFORMULATION OF THE INTERDICTION MODEL

4.1 Preliminary Ideas

We would prefer to convert the model (I-R) into a standard (minimizing or maximizing) mixed-integer program (MIP) because a wealth of techniques exist to solve such models efficiently. (I-R) possesses several features that make this conversion difficult, however:

- (I-R) is as a max-min problem, not a simple minimization or maximization. This difficulty can be overcome by "dualizing" the inner minimization (DC-OPF-R). This converts (I-R) into max-max problem, i.e., a "simple" maximization. However, as we will see later, this conversion leads to other difficulties that must be overcome.

- (I-R) is highly non-linear due to the presence of multiple products of variables associated with the admittance equation (IDC-R.1).

The following two ideas enable us to convert (I-R) into a MIP:

(a) Dropping equality non-linear constraints:

Consider an admittance equation for a generic line, with potential interdiction, represented as a non-linear equality of the form:

$$P = B(\theta_a - \theta_b)(1-\delta_1)(1-\delta_2),$$

along with capacity constraints:

$P \le \overline{P}(1-\delta_1)$ and $P \le \overline{P}(1-\delta_2)$

$P \ge -\overline{P}(1-\delta_1)$ and $P \ge -\overline{P}(1-\delta_2)$.

Here, P, θ_a and θ_b are continuous decision variables representing power flow on the line and phase angles at buses *a*, *b*, respectively; δ_1 and δ_2 are binary decision variables representing two possible ways to interdict the line, say, attacking the line directly and attacking one of the buses the line is connected to.

When both δ_1 and δ_2 are 0, the admittance equation and capacity constraints become: $P = B(\theta_a - \theta_b)$, $P \le \overline{P}$, $P \ge -\overline{P}$, which is the usual power flow admittance equation and capacity constraint. On the other hand, if either δ_1 or δ_2 equals 1, indicating an interdiction has occurred, the above equations become: $P = 0$, $P \le 0$, $P \ge 0$, which is the desired effect—there is no power flow on the line, while θ_a and θ_b may vary independently now.

Remark: It does not suffice to set $P \le 0$ and $P \ge 0$ using the capacity constraints only. The reason is that although this would imply $P = 0$, the admittance constraint would become $0 = B(\theta_a - \theta_b)$, forcing $\theta_a = \theta_b$, which is not necessarily optimal.

To avoid the nonlinearities in the admittance equation, we can establish two constraints that enforce $P = B(\theta_a - \theta_b)$ when all δ-variables are 0, and "drop" this constraint when any of the δ-variables is 1. Let $\overline{\theta}_{ab}$ be an upper bound on the absolute value of the maximum phase angle difference, define: $M = \overline{P} + B\overline{\theta}_{ab}$, then

$$P = B(\theta_a - \theta_b)(1-\delta_1)(1-\delta_2) \equiv \begin{cases} P - B(\theta_a - \theta_b) \le M(\delta_1 + \delta_2) \\ P - B(\theta_a - \theta_b) \ge -M(\delta_1 + \delta_2) \end{cases}. \tag{L.I}$$

Notice that when both δ_1 and δ_2 are 0, the two linear inequalities (L.I) yield precisely $P - B(\theta_a - \theta_b) = 0$. If either δ_1 or δ_2 equals 1, the upper and lower bound limits on the

constraints are sufficiently large that they can never bind, i.e., the original constraints vanish as they should.

(b) Linearizing cross-products:

In the development that follows, we encounter a number of cross-products of the form $\delta\pi$, where δ is a 0-1 variable, representing interdiction, and π is a continuous, non-negative or non-positive variable representing the dual variable for a line capacity constraint like those in (IDC-R.3).

For simplicity, let us assume that $\pi \geq 0$ and that an upper bound $\bar{\pi} \geq \pi$ is known. The $\bar{\pi}$ bound can be established by analyzing the maximum benefit per unit that we could obtain by increasing the line capacity. Assuming that the largest penalty for failing to meet the demand ($\max_{i,c} f_{i,c}$) is greater than the maximum generating cost ($\max_{g} h_g$), a bound that would work in most cases is $\bar{\pi} = \max_{i,c} f_{i,c}$. A more conservative, but generally valid, bound is $\bar{\pi} = 2 \cdot \max_{i,c} f_{i,c} - \min_{i,c} f_{i,c}$.

If we define a new continuous variable $v = \delta\pi$, we can represent the cross product in linear form as:

$$\delta\pi \quad \equiv \begin{cases} v \leq \bar{\pi}\delta \\ v \leq \pi \\ v \geq \pi - \bar{\pi}(1-\delta) \\ 0 \leq \pi \leq \bar{\pi} \\ v \geq 0 \end{cases} \qquad \text{(L.II)}$$

The following diagram shows the validity of this transformation:

$$\left.\begin{array}{l} v \le \bar{\pi}\delta \\ v \le \pi \\ v \ge \pi - \bar{\pi}(1-\delta) \\ 0 \le \pi \le \bar{\pi} \\ v \ge 0 \end{array}\right\} \rightarrow \begin{array}{l} \delta = 0 \Rightarrow \left.\begin{array}{c} v \le 0 \\ v \le \pi \\ v \ge \pi - \bar{\pi} \\ 0 \le \pi \le \bar{\pi} \\ v \ge 0 \end{array}\right\} \Rightarrow \begin{cases} v = 0 \\ \pi \in [0, \bar{\pi}] \end{cases} \\ \delta = 1 \Rightarrow \left.\begin{array}{c} v \le \bar{\pi} \\ v \le \pi \\ v \ge \pi \\ 0 \le \pi \le \bar{\pi} \\ v \ge 0 \end{array}\right\} \Rightarrow \begin{cases} v = \pi \\ \pi \in [0, \bar{\pi}] \end{cases} \end{array}$$

4.2 Linearizing Admittance Equations

Using the linearizing inequalities (L.I), we can convert the admittance equations

$$\boxed{\begin{array}{l} P_{t,l}^{Line} = B_l(\theta_{t,o(l)} - \theta_{t,d(l)})(1-\lambda_l^{Line}\beta_{t,l}^{Line}\delta_l^{Line}) \prod\limits_{i \in I^* | l \in L_i^{Bus}} (1-\beta_{t,i}^{Bus}\delta_i^{Bus}) \cdot \\ \qquad \prod\limits_{s \in S^* | l \in L_s^{Sub}} (1-\beta_{t,s}^{Sub}\delta_s^{Sub}) \prod\limits_{ll \in L^* | ll \in L_l^{Line}} (1-\beta_{t,ll}^{Line}\delta_{ll}^{Line}) \quad \forall l, \forall t \qquad (\pi_{t,l}^{A^-}) \end{array}} \quad \text{(IDC-R.1)}$$

into the following linear expressions:

$$\boxed{\begin{array}{l} P_{t,l}^{Line} - B_l(\theta_{t,o(l)} - \theta_{t,d(l)}) \le M_l(\lambda_l^{Line}\beta_{t,l}^{Line}\delta_l^{Line} + \sum\limits_{i \in I^* | l \in L_i^{Bus}} \beta_{t,i}^{Bus}\delta_i^{Bus} + \sum\limits_{s \in S^* | l \in L_s^{Sub}} \beta_{t,s}^{Sub}\delta_s^{Sub} + \sum\limits_{ll \in L^* | ll \in L_l^{Line}} \beta_{t,ll}^{Line}\delta_{ll}^{Line}) \quad \forall l, \forall t \quad (\pi_{t,l}^{A^-}) \\ P_{t,l}^{Line} - B_l(\theta_{t,o(l)} - \theta_{t,d(l)}) \ge -M_l(\lambda_l^{Line}\beta_{t,l}^{Line}\delta_l^{Line} + \sum\limits_{i \in I^* | l \in L_i^{Bus}} \beta_{t,i}^{Bus}\delta_i^{Bus} + \sum\limits_{s \in S^* | l \in L_s^{Sub}} \beta_{t,s}^{Sub}\delta_s^{Sub} + \sum\limits_{ll \in L^* | ll \in L_l^{Line}} \beta_{t,ll}^{Line}\delta_{ll}^{Line}) \quad \forall l, \forall t \quad (\pi_{t,l}^{A^+}) \end{array}}$$

(IDC-R′.1)

The revised interdiction model with restoration, (I-R′), becomes:

(I-R′): $\max\limits_{\mathbf{\delta}} \min\limits_{P_t^{Gen}, P_t^{Line}, S_t, \theta_t}$ (I-R.0)

subject to:

(I-R.1), (I-R.2)

(IDC-R′.1)

(IDC-R.2) to (IDC-R.6)

Accordingly, the inner power-flow problem can be called (DC-OPR-R′):

(DC-OPF-R′): $\min_{P_t^{Gen}, P_t^{Line}, S_t, \theta_t}$ (I-R.0)

subject to:

(IDC-R′.1)

(IDC-R.2) to (IDC-R.6)

where we assume a given interdiction plan $\delta = \hat{\delta}$.

4.3 Duality: Converting the Model into a Simple Mixed-Integer Program

If we take the dual of the inner model in (I-R′), i.e., the dual of (DC-OPF-R′), we obtain: Model (D-I-R′):

$$\boxed{\begin{aligned}
&\max_{\boldsymbol{\delta}} \max_{\boldsymbol{\pi}} \sum_l M_l \sum_t \Bigg\{ \lambda_l^L \beta_{t,l}^L \delta_l^{Line} \ \ (\pi_{t,l}^{A^-} - \pi_{t,l}^{A^+}) + \sum_{i \in I^* | l \in L_i^{Bus}} \beta_{t,i}^B \delta_i^{Bus} \ \ (\pi_{t,l}^{A^-} - \pi_{t,l}^{A^+}) \\
&+ \sum_{s \in S^* | l \in L_s^{Sub}} \beta_{t,s}^{Sub} \delta_s^{Sub} \ \ (\pi_{t,l}^{A^-} - \pi_{t,l}^{A^+}) + \sum_{ll \in L^* | l \in L_{ll}^{Line}} \beta_{t,ll}^L \delta_{ll}^{Line} \ \ (\pi_{t,l}^{A^-} - \pi_{t,l}^{A^+}) \Bigg\} \\
&+ \sum_{i,t} \Big(\sum_c d_{ic}\Big) \cdot \pi_{t,i}^{Bal} + \sum_t \sum_{g \notin G^{**}} \overline{P}_g^G \pi_{t,g}^{G_0} + \sum_{g \in G^*} \sum_{t | \beta_{t,g}^{Gen}=1} \overline{P}_g^G (1 - \delta_g^{Gen}) \pi_{t,g}^G \\
&+ \sum_{g | i(g) \in I^*} \sum_{t | \beta_{t,i(g)}^{Bus}=1} \overline{P}_g^G (1 - \delta_{i(g)}^{Bus}) \pi_{t,g}^{GB} + \sum_{g | s(i(g)) \in S^*} \sum_{t | \beta_{t,s(i(g))}^{Sub}} \overline{P}_g^G (1 - \delta_{s(i(g))}^{Sub}) \pi_{t,g}^{GS} \\
&+ \sum_t \sum_{l \notin L^{**}} \overline{P}_l^L (\pi_{t,l}^{L_0^-} - \pi_{t,l}^{L_0^+}) + \sum_{l \in L^*} \sum_{t | \beta_{t,l}^{Line}=1} \overline{P}_l^L (1 - \delta_l^{Line}) \ (\pi_{t,l}^{L^-} - \pi_{t,l}^{L^+}) \\
&+ \sum_{i \in I^* | l \in L_i^{Bus}} \sum_{t | \beta_{t,i}^{Bus}=1} \overline{P}_l^L (1 - \delta_i^{Bus}) \ (\pi_{t,l,i}^{LB^-} - \pi_{t,l,i}^{LB^+}) + \sum_{s \in S^* | l \in L_s^{Sub}} \sum_{t | \beta_{t,s}^{Sub}=1} \overline{P}_l^L (1 - \delta_s^{Sub}) \ (\pi_{t,l,s}^{LS^-} - \pi_{t,l,s}^{LS^+}) \\
&+ \sum_{ll \in L^* | l \in L_{ll}^{Line}} \sum_{t | \beta_{t,ll}^{Line}=1} \overline{P}_l^L (1 - \delta_{ll}^{Line}) \ (\pi_{t,l,ll}^{LL^-} - \pi_{t,l,ll}^{LL^+}) + \sum_{i,c,t} d_{i,c} \ \ \pi_{t,i,c}^{Load}
\end{aligned}}$$

(D-I-R′.0)

subject to:

- Dual constraints for power generation:

$$\boxed{\pi_{t,i(g)}^{Bal} + (1 - \lambda_{t,g}^{G_0}) \pi_{t,g}^{G_0} + \lambda_g^G \pi_{t,g}^G + \lambda_{i(g)}^I \beta_{t,i(g)}^{Bus} \pi_{t,g}^{GB} + \lambda_{s(i(g))}^S \beta_{t,s(i(g))}^{Bus} \pi_{t,g}^{GS} \le D(t) h_g, \ \forall t, g \quad (P_{t,g}^G)}$$

(D-I-R′.1)

- Dual constraints for power flow on lines:

$$\boxed{\begin{aligned}&\pi_{t,l}^{A^-}+\pi_{t,l}^{A^+}+(1-\lambda_{t,l}^{L_0})(\pi_{t,l}^{L_0^-}+\pi_{t,l}^{L_0^+})+\lambda_l^L\beta_{t,l}^{Line}\pi_{t,l}^{LCap^-}+\lambda_l^L\beta_{t,l}^{Line}\pi_{t,l}^{LCap^+}+\sum_{i\in I^*|l\in L_i^{Bus},\beta_{t,i}^{Bus}=1}\left(\pi_{t,l,i}^{LB^-}+\pi_{t,l,i}^{LB^+}\right)\\&+\sum_{s\in S^*|l\in L_s^{Sub},\beta_{t,s}^{Sub}=1}\left(\pi_{t,l,s}^{LS^-}+\pi_{t,l,s}^{LS^+}\right)+\sum_{ll\in L^*|ll\in L_l^{Par},\beta_{t,ll}^{Line}=1}\left(\pi_{t,l,ll}^{LL^-}+\pi_{t,l,ll}^{LL^+}\right)-\pi_{t,o(l)}^{Bal}+\pi_{t,d(l)}^{Bal}=0,\quad\forall\ t,l\ \ (P_{t,l}^L)\end{aligned}}$$

(D-I-R′.2)

- Dual constraints for power shedding:

$$\boxed{\pi_{t,i}^{Bal}+\pi_{t,i,c}^{Load}\le D(t)\cdot f_{ic},\qquad\forall\ t,i,c\quad(\mathrm{S}_{\mathrm{t,i,c}})}\tag{D-I-R′.3}$$

- Dual constraints for phase angles:

$$\boxed{-\sum_{l|o(l)=i}B_l\left(\pi_{t,l}^{A^-}+\pi_{t,l}^{A^+}\right)+\sum_{l|d(l)=i}B_l\left(\pi_{t,l}^{A^-}+\pi_{t,l}^{A^+}\right)=0,\quad\forall\ t,i\quad(\theta_{\mathrm{t,i}})}\tag{D-I-R′.4}$$

- Dual variable sign:

$$\boxed{\begin{aligned}&\pi^{Bal}\ \ \text{unrestricted}\\&\pi^{A^-},\pi^{\mathrm{L}_0^-},\pi^{LCap^-},\ \pi^{LB^-},\pi^{LS^-},\pi^{LL^-}\le 0\\&\pi^{A^+},\pi^{\mathrm{L}_0^+},\pi^{LCap^+},\ \pi^{LB^+},\pi^{LS^+},\pi^{LL^+}\ge 0\\&\pi^{\mathrm{G}_0},\pi^{G},\pi^{GB},\ \pi^{GS}\le 0\\&\pi^{Load}\le 0\end{aligned}}\tag{D-I-R′.5}$$

- Interdiction resource (same as (I-R.1) and (I-R.2)):

$$\boxed{\sum_{g\in\mathrm{G}^*}M_g^{Gen}\,\delta_g^{Gen}+\sum_{l\in\mathrm{L}^*}M_l^{Line}\,\delta_l^{Line}+\sum_{i\in\mathrm{I}^*}M_i^{Bus}\,\delta_i^{Bus}+\sum_{s\in\mathrm{S}^*}M_s^{Sub}\,\delta_s^{Sub}\le M}\tag{I-R.1}$$

$$\boxed{\text{All }\delta\text{-variables are binary}}\tag{I-R.2}$$

In outline, this model is:

$$\text{(D-I-R}'\text{):} \qquad \max_{\boldsymbol{\delta}} \min_{\boldsymbol{\pi}} \text{(D-I-R}'\text{.0)}$$

subject to:

(I-R.1), (I-R.2)

(D-I-R′.1) to (D-I-R′.5)

4.4 Linearizing Cross-products in the Objective Function

The objective function (D-I-R′.0) contains cross-products of the form $\delta\pi$. Using the linearizing technique (L.II) from Section 4.1, we can linearize the objective function as follows:

$$\boxed{\begin{aligned}
&\max_{\delta} \max_{\pi,v} \sum_{l} M_l \Bigg\{ \lambda_l^L \beta_{t,l}^{Line} (v_{t,l}^{A^-} - v_{t,l}^{A^+}) + \sum_{i \in I^* | l \in L_i^{Bus}} \beta_{t,i}^{Bus} (v_{t,l}^{BA^-} - v_{t,l}^{BA^+}) \\
&+ \sum_{s \in S^* | l \in L_s^{Sub}} \beta_{t,s}^{Sub} (v_{t,l,s}^{SA^-} - v_{t,l,s}^{SA^+}) + \sum_{ll \in L^* | l \in L_{ll}^{Line}} \beta_{t,ll}^{Line} (v_{t,l,ll}^{LA^-} - v_{t,l,ll}^{LA^+}) \Bigg\} \\
&+ \sum_{i,t} \Big(\sum_{c} d_{ic} \Big) \cdot \pi_{t,i}^{Bal} + \sum_{t} \sum_{g \notin G^{**}} \overline{P}_g^G \pi_{t,g}^{G_0} + \sum_{g \in G^*} \sum_{t | \beta_{t,g}^{Gen} = 1} \overline{P}_g^G (\pi_{t,g}^{G} - v_{t,g}^{G}) \\
&+ \sum_{g | i(g) \in I^*} \sum_{t | \beta_{t,i(g)}^{Bus} = 1} \overline{P}_g^G (\pi_{t,g}^{GB} - v_{t,g}^{GB}) + \sum_{g | s(i(g)) \in S^*} \sum_{t | \beta_{t,s(i(g))}^{Sub} = 1} \overline{P}_g^G (\pi_{t,g}^{GS} - v_{t,g}^{GS}) \\
&+ \sum_{t} \sum_{l \notin L^{**}} \overline{P}_l^L (\pi_{t,l}^{L_0^-} - \pi_{t,l}^{L_0^+}) + \sum_{l \in L^*} \sum_{t | \beta_{t,l}^{Line} = 1} \overline{P}_l^L \Big((\pi_{t,l}^{L^-} - \pi_{t,l}^{L^+}) - (v_{t,l}^{L^-} - v_{t,l}^{L^+}) \Big) \\
&+ \sum_{i \in I^* | l \in L_i^{Bus}} \sum_{t | \beta_{t,i}^{Bus} = 1} \overline{P}_l^L \Big((\pi_{t,l,i}^{LB^-} - \pi_{t,l,i}^{LB^+}) - (v_{t,l,i}^{LB^-} - v_{t,l,i}^{LB^+}) \Big) \\
&+ \sum_{s \in S^* | l \in L_s^{Sub}} \sum_{t | \beta_{t,s}^{Sub} = 1} \overline{P}_l^L \Big((\pi_{t,l,s}^{LS^-} - \pi_{tl,s}^{LS^+}) - (v_{t,l,s}^{LS^-} - v_{t,l,s}^{LS^+}) \Big) \\
&+ \sum_{ll \in L^* | l \in L_{ll}^{Line}} \sum_{t | \beta_{t,ll}^{Line} = 1} \overline{P}_l^L \Big((\pi_{t,l,ll}^{LL^-} - \pi_{t,l,ll}^{LL^+}) - (v_{t,l,ll}^{LL^-} - v_{t,l,ll}^{LL^+}) \Big) + \sum_{i} \sum_{c} \sum_{t} d_{ic} \, \pi_{t,i,c}^{Load}
\end{aligned}} \qquad \text{(D-I-R}''\text{.0)}$$

In addition, we require a set of constraints of the form:

$$\boxed{\begin{array}{l} v \le \bar{\pi}\delta \\ v \le \pi \\ v \ge \pi - \bar{\pi}(1-\delta) \\ 0 \le \pi \le \bar{\pi} \\ v \ge 0 \end{array}} \qquad \text{(D-I-R}''\text{.1)}$$

for every linearized cross-product. These constraints are specified, in detail, in Appendix B.

The final model, in outline form, is:

(D-I-R″): $\max_{\delta,\pi,v}$ (D-I-R″.0)

subject to:

(I-R.1), (I-R.2)

(D-I-R′.1) to (D-I-R′.5)

(D-I-R″.1)

Model (D-I-R″) is the culmination of all the linearizations and dualizations described previously. It exhibits multiple advantages: The most important one is to represent the interdiction problem as standard, compact, mixed-integer program (MIP). That means that any of the generic optimization techniques available for solving, bounding, or approximating the solution to a MIP are applicable to this model.

Hereafter, we refer to model (D-I-R″) as (I-MIP), i.e., "Interdiction problem in MIP form."

5. TEST CASES

We present a summary of results to show:

(a) the benefit of incorporating system restoration into our analysis, and

(b) the potential of the MIP reformulation, which can be solved exactly.

For item (a), we will compare the enhanced heuristic with system restoration (I-ALG-R) with the former heuristic (I-ALG). For item (b), we will compare the heuristic solution provided by (I-ALG-R) with the exact solution obtained by solving the MIP reformulation (I-MIP).

Our tests are carried out on the same set of IEEE reliability test networks ("One" and "Two" Areas) described in our previous report. Assumptions regarding outage duration are summarized in the following table:

Grid Component	**Interdictable**	**Resources *M* (no. of terrorists)**	**Outage Duration (h)**
Lines (overhead)	YES	1	72
Lines (underground)	NO	N/A*	N/A*
Transformers	YES	2	768
Buses	YES	3	360
Generators	NO	N/A*	N/A*
Substations	YES	3	768

*Not Applicable.

Data for outage durations (i.e., repair or replacement times) are based loosely on IEEE [1999-I]. Outage duration for transformers is 768 hours. For overhead lines, instead of the 10 or 11 hours used in IEEE [1999-I], we are more conservative and assume 72 hours. This is justified because (a) we expect more damage to result from the intentional destruction of a line—this would probably involve the destruction of one or more towers [*Miami Herald*, 2002]—than the average time needed to repair damage from common natural causes such as lightning, and (b) if *n* lines and other grid elements are attacked, total repair time may be longer if fewer than *n* repair teams are available. We also assume that a large substation requires 768 hours for repair, but buses, for which IEEE [1999-I] provides no data, require 360 hours.

5.1 Comparing Solutions with and without System Restoration

Although in both cases below I-ALG finds better short-term disruptions (compare power shed) over the first 72 hours following the attack, it is clear that long-term effects are better captured by I-ALG-R:

Case/Algorithm	**Directly Interdicted Components. Resource: M=6**	**Time Period**	**Power Shed (MW)**	**Energy Shed (MWh)**
RTS-One-Area I-ALG	Lines: A11, A20, A21, A25-1, A27, A33-1	0-72 h	1,373	98,856
				Total: 98,856
RTS-One-Area I-ALG-R	Lines: A23 Transformers: A7 Substations: Sub-A2	0-72 h	902	64,944
		72-768 h	708	492,768
				Total: 557,712

Case/Algorithm	Directly Interdicted Components. Resource: M=12	Time Period	Power Shed (MW)	Energy Shed (MWh)
RTS-Two-Areas I-ALG	Lines: A20, A21, A27, A33-1, A35-1, AB2, B20, B21, B25-1, B27, B33-1, B34	0-72 h	2,516	181,152
				Total: 181,152
RTS-Two-Areas I-ALG-R	Substations: Sub-A1, Sub-A2, Sub-B1, Sub-B2	0-768	1,416	1,087,488
				Total: 1,087,488

Although our goal is focused on long-term disruption, we recognize that the optimal solution provided by the I-ALG model is still an insightful measure of vulnerability in the analysis of short-term effects.

5.2 Comparing Heuristic and Optimal Solutions

First Goal: Optimal short-term disruption. In this case we are comparing I-ALG with a special version of the I-MIP model, in which there is only one period "t = 1," the duration of this period is $D_1 = 1$ hour, and the duration outage for all interdictable elements $e \in \xi$ is $Dur(e) = 1$ hour.

From the next two tables, the heuristic solution for optimal short-term disruption (provided by I-ALG) is of good quality when compared to the best possible provided by (I-MIP):

Case/Algorithm	Directly Interdicted Components. Resource: M=6	Power Shed (MW)
RTS-One-Area I-ALG	Lines: A11, A20, A21, A25-1, A27, A33-1	1,373
RTS-One-Area I-MIP	Lines: A11, A20, A21, A25-2, A27, A33-1	1,373

Case/Algorithm	Directly Interdicted Components. Resource: M=12	Power Shed (MW)
RTS-Two-Areas I-ALG	Lines: A20, A21, A27, A33-1, A35-1, AB2, B20, B21, B25-1, B27, B33-1, B34	2,516
RTS-Two-Areas I-MIP	Lines: A21, A25-1, A27, A33-1, B18, B21, B25-1, B27, B33-2	2,781

However, this quality deteriorates when interdiction resource increases:

Case/Algorithm	Directly Interdicted Components. Resource: M=24	Power Shed (MW)
RTS-Two-Areas I-ALG	Buses: 118, 123, 216, 217 Substations: Sub-A1, Sub-A2, Sub-B1, Sub-B2	3,266
RTS-Two-Areas I-MIP	Lines: A21, A33-1, A34, B21, B27, B33-1 Buses: 113, 115, 118, 213, 215, 218	4,142

This shows the MIP optimal solution may outperform the solution provided by the heuristic algorithm by 25% in some cases.

Second Goal: Optimal long-term disruption. In this case, we are comparing I-ALG-R with the I-MIP configured for a total horizon of 768 hours, which is the longer time to repair for any component in our test cases.

Again, the first two tables below show that the heuristic solution for optimal long-term disruption (provided by I-ALG-R) is acceptable for the cases tested.

Case/Algorithm	Directly Interdicted Components. Resource: M=6	Time Period	Power Shed (MW)	Energy Shed (MWh)
RTS-One-Area I-ALG-R	Lines: A23 Transformers: A7 Substations: Sub-A2	0-72 h	902	64,944
		72-768 h	708	492,768
				Total: 557,712
RTS-One-Area I-MIP	Lines: A23 Transformers: A7 Substations: Sub-A2	0-72 h	902	64,944
		72-768 h	708	492,768
				Total: 557,712

Case/Algorithm	Directly Interdicted Components. Resource: M=12	Time Period	Power Shed (MW)	Energy Shed (MWh)
RTS-Two-Areas I-ALG-R	Substations: Sub-A1, Sub-A2, Sub-B1, Sub-B2	0-768	1,416	1,087,488
				Total: 1,087,488
RTS-Two-Areas I-MIP	Lines: A23, B23 Transformers: A7, B7 Substations: Sub-A2, Sub-B2	0-72 h	1,804	129,888
		72-768 h	1,416	985,536
				Total: 1,115,424

In the case below, the solution provided by the heuristic for a problem with a larger interdiction resource value still exhibits acceptable quality.

Case/Algorithm	Directly Interdicted Components. Resource: M=24	Time Period	Power Shed (MW)	Energy Shed (MWh)
RTS-Two-Areas I-ALG-R	Buses: 116, 118, 215, 218 Substations: Sub-A1, Sub-A2, Sub-B1, Sub-B2	0-360 h	2,693	969,480
		360-768 h	1,416	577,728
				Total: 1,547,208
RTS-Two-Areas I-MIP	Lines: A30, A33-2 Transformers: A7, B7 Buses: 115, 118, 215, 218 Substations: Sub-A2, Sub-B2	0-72 h	3,164	227,808
		72-360 h	2,716	782,208
		360-720 h	1,416	577,728
				Total: 1,587,744

6. Vulnerability of Electric Power Grid Analyzer (VEGA)

6.1 Overview

VEGA is an integrated decision-support system comprising a GUI, a relational database management system (RDBMS), an optimization module, and an administration program that

controls all of those components (Figure 3). VEGA 1.0 is the first version of this system and has been built on the Microsoft (MS) Windows 2000 operating system [Microsoft 2003]. This section provides an overview of VEGA.

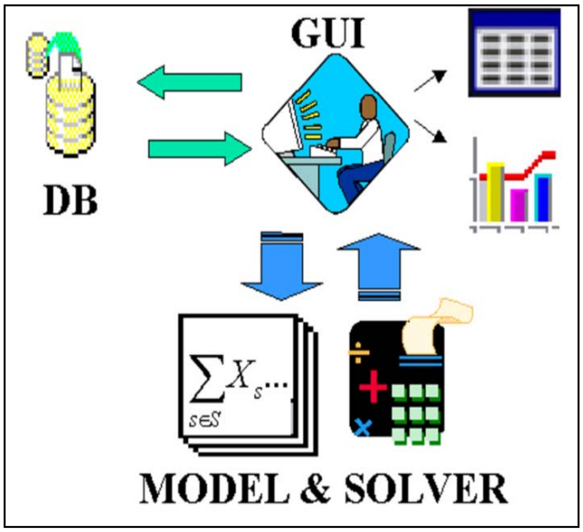

Figure 3. VEGA decision support system.

The ultimate goals of VEGA are to enable access to our analytical techniques for a wide range of potential users, including students in the HSLD curriculum, and to bridge the gap between the underlying mathematical optimization methods and the decision-makers. A preliminary Web page has been set up for this project [VEGA, 2003].

The GUI and database (DB) are key to organizing planning data, reducing clerical error through embedded validations, completing missing details, filtering information according to user's needs, and displaying multiple scenarios with their results, for comparison purposes. The GUI is also key for demonstrating the potential that optimization techniques have for planning interdiction and interdiction defense.

The GUI helps prepare power-network data for analysis, and then displays analytical results, by enabling easy navigation through customized tables and graphics containing problem data and results. This gives a user easy access to the mathematical analysis of a problem even if the user in not an expert in mathematical modeling and optimization.

The VEGA optimization module performs the mathematical analysis of the problem independently of the GUI. The purpose of the GUI is to prepare the case data to be analyzed and to retrieve and display the optimization results in a user-friendly fashion. VEGA 1.0's core is an optimization model that assesses the maximum possible disruption a network might experience from a terrorist attack. Naturally, this core can work as independent entity, and its operation is, in fact, transparent to the user of VEGA.

The administration program and the GUI are implemented in the MS Visual Basic (VB) 6.0 programming language [Microsoft, 1998], supported by a RDBMS implemented with MS Access 2000 [Microsoft, 2003]. The underlying optimization module is implemented using GAMS [GAMS, 2003, Brooke et al., 1996]. Data transfer and synchronization of the GUI with GAMS are performed by means of plain ASCII files, because GAMS is not available as a callable or dynamic library; therefore, GAMS executes as an external program.

6.2 VEGA GUI Overview

The front-end application responsible for the VEGA GUI uses a Windows-based methodology that facilitates for the user:

a. Network data input;

b. Other data input, including possible scenarios, optimization parameters, etc.;

c. Analyzing results provided by the optimization model;

d. Graphical display of the network, input data, and output results; and

e. Administration of multiple cases with several scenarios per case.

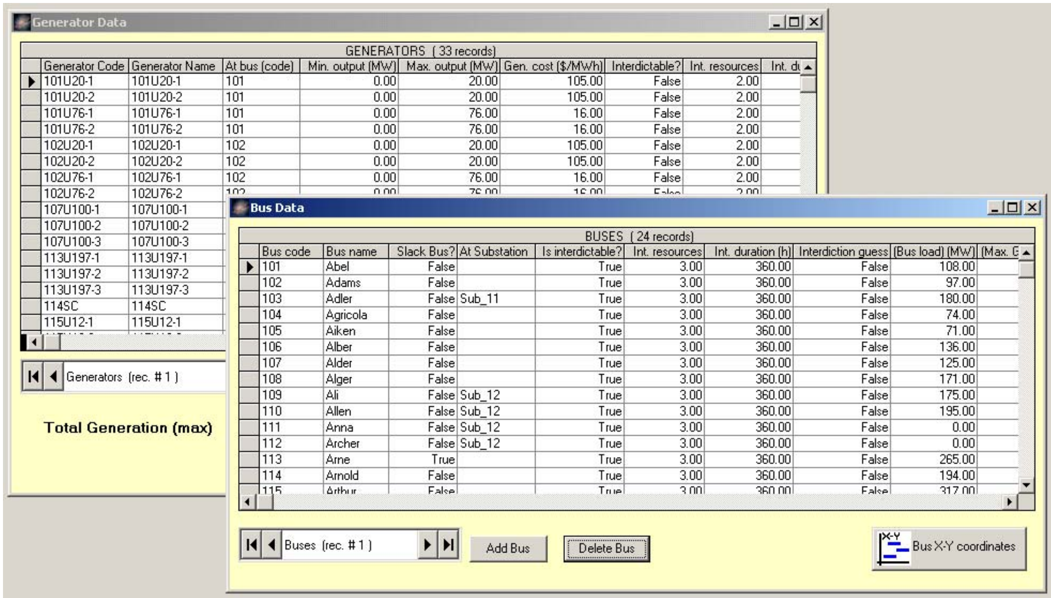

Figure 4. Examples of VEGA data tables.

The GUI in VEGA 1.0 uses VB tables in order to import data from the database, and edit the records associated with a problem. Figure 4 shows an example of data tables for Buses and Generators.

Upon completing necessary data entry, the user can invoke the optimization module to produce optimal, or near-optimal, interdiction plans. These plans can be displayed in tabular form (Figure 5) or in graphical form (Figure 6).

Figure 5. Interdiction plan shown in tabular form.

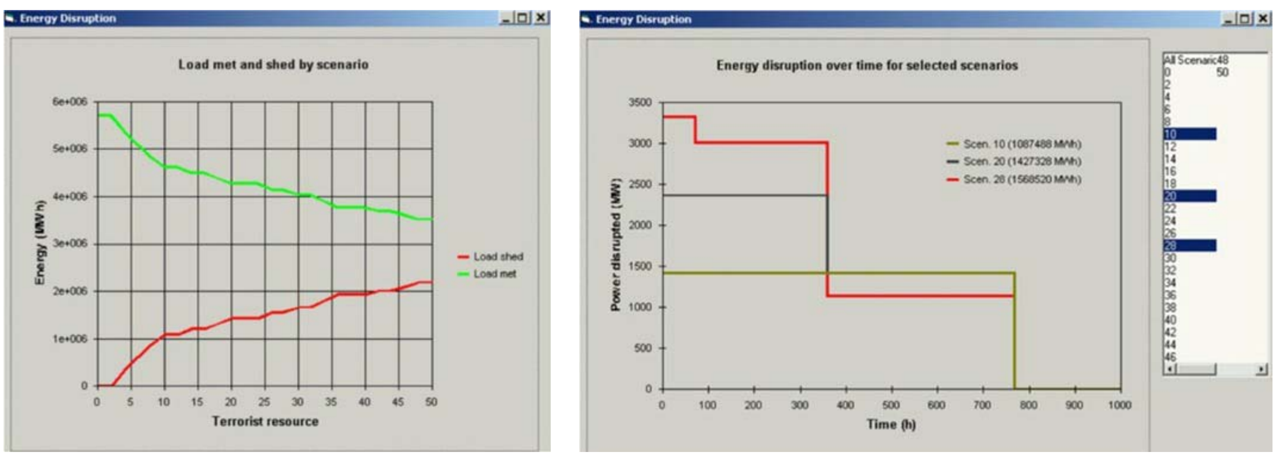

Figure 6. Graphical representation of overall results by scenario (left) and by scenario and time (right).

One important, recent accomplishment has been the enhancement of a module in the VEGA GUI called the "One-Line Diagram (OD) GUI." ODs are used by electrical engineers to represent electric power grids graphically. The VEGA 1.0 (see VEGA [2003]) OD GUI had many limitations that have been overcome through the thesis research of an NPS student [Stathakos, 2003]. Figure 7 shows one of the new OD representations.

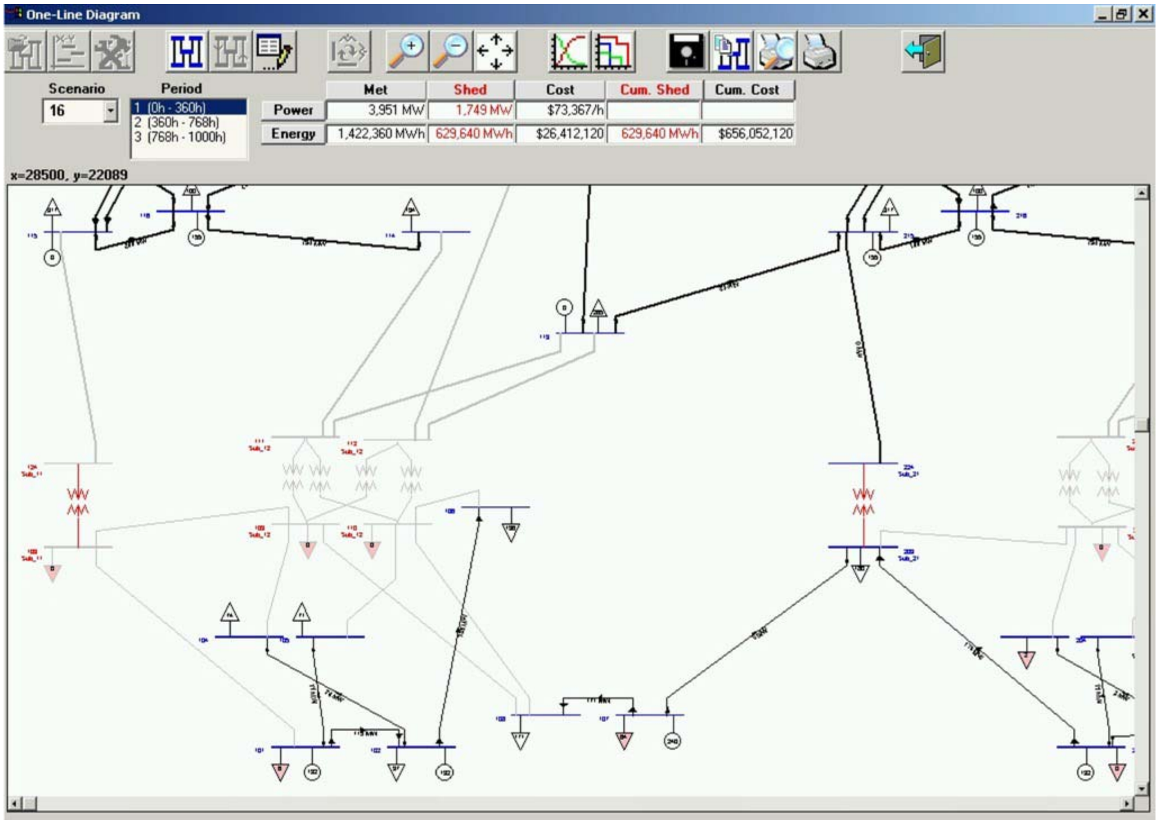

Figure 7. One-Line Diagram for an electric network under interdiction.

The above snapshots are intended to give an overview of the VEGA GUI, only, and a full report on its capabilities will be provided in a separate, future report.

7. OTHER ACTIVITIES

7.1 Thesis Students

To date, two students have devoted their Master's thesis research to our project, Major Dimitrios Sthatakos (Greek Army) and LCDR Roger Alvarez (USN).

Major Sthatakos, who graduated in December 2003, enhanced the one-line diagram (OD) interface in our VEGA GUI. The OD GUI represents the details of power flows and interdictions graphically (Figure 7). Major Sthatakos created a highly flexible OD GUI by replacing a prototypic OD GUI (based on standard Visual Basic objects) with an advanced OD GUI based on state-of-the-art ActiveX controls. For more information, see Sthatakos [2003].

LCDR Alvarez is currently working on the MIP representation of our mathematical models (see Section 4) and on their solvability. He has been instrumental in completing many of the refinements to the MIP described in this report, and he is exploring alternative solution techniques for the MIP, with special focus on Benders decomposition. He is expected to graduate in March 2004 [Alvarez, 2004].

We continue to seek the involvement of NPS students in our project.

7.2 Other Reports and Activities

In addition to our previous report [Salmeron, Wood, and Baldick, 2003-I], our first year's work has yielded a refereed publication: "Analysis of Electric Grid Security Under Terrorist Threat," [Salmeron, Wood, and Baldick, 2003-II], which will appear in *IEEE Transactions on Power Systems*. The feedback from the four reviewers was highly positive.

Also, our work was presented in the CS3660 seminar (Critical Infrastructure Protection) as part of the recently created HSLD curriculum at NPS.

7.3 Other Sources of Funding

The Department of Justice has been our sole source of funding to date. We are seeking additional research support from the Department of Energy (proposal submitted) and the National Science Foundation (proposal in preparation).

8. FUTURE WORK

The major challenge we face in the present year is to obtain actual U.S. power-grid data for testing and validating our methodology. In doing so, we need to continue the development of techniques to solve the exact models (MIPs) for those cases and other realistically sized problems. (See the proposal, Salmeron, and Wood [2003], for more detail.) Currently, we are acquiring data sets for different areas in the U.S. North American Electric Reliability Council system, and we are adapting and extending those data for our purposes. We have had success in formulating our models as MIPs, but additional development and experimentation is needed in order to be able to solve them efficiently. In particular, we will be investigating special bounding techniques and specialized cutting-plane techniques (Geoffrion, [1972], Israeli and Wood [2002]).

During the coming year, we will also:

- Continue to work on the VEGA GUI;
- Extend our models and algorithms to capture the dynamics of load variation over time. This requires extension of the VEGA optimization module, as well as the VEGA database and GUI (at the levels of data and result management and of graphical representations); and
- Initiate work on trilevel models to identify optimal protection plans for power grids.

9. VALUE OF THE RESEARCH TO HOMELAND SECURITY

The call for proposals that this research addresses, asks how our research adds value to the Homeland Security effort. We respond as follows:

Simulation software for Homeland Security (HLS):

- Attacks on critical infrastructure: Power Grids

Deliverables (this document):

- Models and algorithms (as presented)
- Case studies (as presented)
- Software (optimization algorithms and GUI under development)
- Publications (as presented)

By criterion used to fund the project:

- This research addresses an important problem in HLS
- This research adds to the body of HLS knowledge
- This research is interdisciplinary
- This research is novel and useful
- This research invites non-NPS collaborators
- Principal Investigators (PIs) have a reputation in the proposed field of study
- PIs will try to get students involved in this research and produce theses
- Results will be publishable
- Results will be useful in teaching HLS courses
- PIs believe the budget is in line with the results

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APPENDIX A: SUMMARY OF PREVIOUS NOTATION

Indices and index sets:

$i \in \mathbf{I}$, buses
$g \in \mathbf{G}$, generating units
$l \in \mathbf{L}$, transmission lines
$c \in \mathbf{C}$, consumer sectors
$s \in \mathbf{S}$, substations
$i \in \mathbf{I}_s$, buses at substation s
$g \in \mathbf{G}_i$, generating units connected to bus i
$l \in \mathbf{L}_i^{Bus}$, lines connected to bus i
$l \in \mathbf{L}_s^{Sub}$, lines connected to substation s (including transformers, which are represented by lines)
$l' \in \mathbf{L}_l^{Par}$, lines $l' \neq l$ running in parallel to line l
$\mathbf{G}^* \subseteq \mathbf{G}$, $\mathbf{L}^* \subseteq \mathbf{L}$, $\mathbf{I}^* \subseteq \mathbf{I}$, $\mathbf{S}^* \subseteq \mathbf{S}$, interdictable generators, lines, buses, and substations, respectively. These are "interdictable components."

Parameters (units):

$o(l)$, $d(l)$, origin and destination buses of line l; more than one line with the same $o(l)$, $d(l)$ may exist
$i(g)$, bus for generator g, i.e., $g \in \mathbf{G}_{i(g)}$
$s(i)$, substation for bus i, for $i \in I \mid \exists s \in S$ where $i \in I_s$
d_{ic}, load of consumer sector c at bus i (MW)
$\overline{P}_l^{Line}$, transmission capacity for line l (MW)
$\overline{P}_g^{Gen}$, maximum output from generator g (MW)
r_l, x_l, resistance, reactance of line l (Ω). (We assume $x_l >> r_l$.); series susceptance is $B_l = x_l / (r_l^2 + x_l^2)$
h_g, generation cost for unit g (\$/MWh)
f_{ic}, load-shedding cost for customer sector c at bus i (\$/MWh)
M_g^{Gen}, M_l^{Line}, M_i^{Bus}, M_s^{Sub}, resource required to interdict generator g, line l, bus i, and substation s, respectively.
M, total interdiction resource available to terrorists.

Decision variables (units):

P_g^{Gen}, generation from unit g (MW)
P_l^{Line}, power flow on line l (MW)
S_{ic}, load shed by customer sector c at bus i (MW)
θ_i, phase angle at bus i (radians)
$\delta_g^{Gen}, \delta_l^{Line}, \delta_i^{Bus}, \delta_s^{Sub}$, binary variables that take the value 1 if generator g, line l, bus i or substation s, respectively, are interdicted, and are 0 otherwise.

APPENDIX B: LINEARIZATION OF CROSS-PRODUCTS

Explicit list of constraints for equations (D-I-R″.1) in model (D-I-R″):

$$\boxed{\begin{aligned}
&v_{t,l}^{A^+} \le \overline{\pi}_{t,l}^{A}\delta_l^{Line}, \ \ \forall t,l \mid l \in L^*, t \in T, \beta_{t,l}^{Line} = 1 \quad (\gamma_{(t,l)_1}^{A^+})\\
&v_{t,l}^{A^+} - \pi_{t,l}^{A^+} \le 0, \ \ \forall t,l \mid l \in L^*, t \in T, \beta_{t,l}^{Line} = 1 \quad (\gamma_{(t,l)_2}^{A^+})\\
&v_{t,l}^{A^+} - \pi_{t,l}^{A^+} \ge -\overline{\pi}_{t,l}^{A}(1-\delta_l^{Line}), \ \ \forall t,l \mid l \in L^*, t \in T, \beta_{t,l}^{Line} = 1 \quad (\gamma_{(t,l)_3}^{A^+})
\end{aligned}}$$

$$\boxed{\begin{aligned}
&v_{t,l,i}^{BA^+} \le \overline{\pi}_{t,l}^{A}\delta_i^{Bus}, \ \ \forall t,i,l \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus} = 1 \ \ (\gamma_{(t,l,i)_1}^{BA^+})\\
&v_{t,l,i}^{BA^+} - \pi_{t,l}^{A^+} \le 0, \ \ \forall t,i,l \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus} = 1 \ \ (\gamma_{(t,l,i)_2}^{BA^+})\\
&v_{t,l,i}^{BA^+} - \pi_{t,l}^{A^+} \ge -\overline{\pi}_{t,l}^{A}(1-\delta_i^{Bus}), \ \ \forall t,i,l \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus} = 1 \ \ (\gamma_{(t,l,i)_3}^{BA^+})
\end{aligned}}$$

$$\boxed{\begin{aligned}
&v_{t,l,s}^{SA^+} \le \overline{\pi}_{t,l}^{A}\delta_s^{Sub}, \ \ \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_1}^{SA^+})\\
&v_{t,l,s}^{SA^+} - \pi_{t,l}^{A^+} \le 0, \ \ \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_2}^{SA^+})\\
&v_{t,l,s}^{SA^+} - \pi_{t,l}^{A^+} \ge -\overline{\pi}_{t,l}^{A}(1-\delta_s^{Sub}), \ \ \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_3}^{SA^+})
\end{aligned}}$$

$$\boxed{\begin{aligned}
&v_{t,l,ll}^{LA^+} \le \overline{\pi}_{t,l}^{A}\delta_{ll}^{Line}, \ \ \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{Line} = 1 \quad (\gamma_{(t,l,ll)_1}^{LA^+})\\
&v_{t,l,ll}^{LA^+} - \pi_{t,l}^{A^+} \le 0, \ \ \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{Line} = 1 \quad (\gamma_{(t,l,ll)_2}^{LA^+})\\
&v_{t,l,ll}^{LA^+} - \pi_{t,l}^{A^+} \ge -\overline{\pi}_{t,l}^{A}(1-\delta_{ll}^{Line}), \ \ \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{Line} = 1 \quad (\gamma_{(t,l,ll)_3}^{LA^+})
\end{aligned}}$$

$$\boxed{\begin{aligned}
&v_{t,l}^{L^+} \le \overline{\pi}_{t,l}^{L}\delta_l^{Line}, \ \ \forall t,l \in L^*, t \in T, \beta_{t,l}^{Line} = 1 \quad (\gamma_{(t,l)_1}^{L^+})\\
&v_{t,l}^{L^+} - \pi_{t,l}^{L^+} \le 0, \ \ \forall t,l \in L^*, t \in T, \beta_{t,l}^{Line} = 1 \quad (\gamma_{(t,l)_2}^{L^+})\\
&v_{t,l}^{L^+} - \pi_{t,l}^{L^+} \ge -\overline{\pi}_{t,l}^{L}(1-\delta_l^{Line}), \ \ \forall t,l \in L^*, t \in T, \beta_{t,l}^{Line} = 1 \quad (\gamma_{(t,l)_3}^{L^+})
\end{aligned}}$$

$$\boxed{\begin{array}{l}
v_{t,l,i}^{LB^+} \le \overline{\pi}_{t,l}^{L}\delta_i^{Bus}, \quad \forall t,l,i \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus}=1 \quad (\gamma_{(t,l,i)_1}^{LB^+}) \\
v_{t,l,i}^{LB^+} - \pi_{t,l,i}^{LB^+} \le 0, \quad \forall t,l,i \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus}=1 \quad (\gamma_{(t,l,i)_2}^{LB^+}) \\
v_{t,l,i}^{LB^+} - \pi_{t,l,i}^{LB^+} \ge -\overline{\pi}_{t,l}^{L}(1-\delta_i^{Bus}), \quad \forall t,l,i \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus}=1 \quad (\gamma_{(t,l,i)_3}^{LB^+})
\end{array}}$$

$$\boxed{\begin{array}{l}
v_{t,l,s}^{LS^+} \le \overline{\pi}_{t,l}^{L}\delta_s^{Sub}, \quad \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_1}^{LS^+}) \\
v_{t,l,s}^{LS^+} - \pi_{t,l,s}^{LS^+} \le 0, \quad \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_2}^{LS^+}) \\
v_{t,l,s}^{LS^+} - \pi_{t,l,s}^{LS^+} \ge -\overline{\pi}_{t,l}^{L}(1-\delta_s^{Sub}), \quad \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_3}^{LS^+})
\end{array}}$$

$$\boxed{\begin{array}{l}
v_{t,l,ll}^{LL^+} \le \overline{\pi}_{t,l}^{L}\delta_{ll}^{Line}, \quad \forall t,l,ll \mid ll \in L^*, \mathrm{ll} \in \mathrm{L}_1^{\mathrm{Par}}, t \in T, \beta_{t,ll}^{Line}=1 \quad (\gamma_{(t,l,ll)_1}^{LL^+}) \\
v_{t,l,ll}^{LL^+} - \pi_{t,l,ll}^{LL^+} \le 0, \quad \forall t,l,ll \mid ll \in L^*, \mathrm{ll} \in \mathrm{L}_1^{\mathrm{Par}}, t \in T, \beta_{t,ll}^{Line}=1 \quad (\gamma_{(t,l,ll)_2}^{LL^+}) \\
v_{t,l,ll}^{LL^+} - \pi_{t,l,ll}^{LL^+} \ge -\overline{\pi}_{t,l}^{L}(1-\delta_{ll}^{Line}), \quad \forall t,l,ll \mid ll \in L^*, \mathrm{ll} \in \mathrm{L}_1^{\mathrm{Par}}, t \in T, \beta_{t,ll}^{Line}=1 \quad (\gamma_{(t,l,ll)_3}^{LL^+})
\end{array}}$$

$$\boxed{\begin{array}{l}
v_{t,l}^{A^-} \ge -\overline{\pi}_{t,l}^{A}\delta_l^{Line}, \quad \forall t,l \mid l \in L^*, t \in T, \beta_{t,l}^{Line}=1 \quad (\gamma_{(t,l)_1}^{A^-}) \\
v_{t,l}^{A^-} - \pi_{t,l}^{A^-} \ge 0, \quad \forall t,l \mid l \in L^*, t \in T, \beta_{t,l}^{Line}=1 \quad (\gamma_{(t,l)_2}^{A^-}) \\
v_{t,l}^{A^-} - \pi_{t,l}^{A^-} \le \overline{\pi}_{t,l}^{A}(1-\delta_l^{Line}), \quad \forall t,l \mid l \in L^*, t \in T, \beta_{t,l}^{Line}=1 \quad (\gamma_{(t,l)_3}^{A^-})
\end{array}}$$

$$\boxed{\begin{array}{l}
v_{t,l,i}^{BA^-} \ge -\overline{\pi}_{t,l}^{A}\delta_i^{Bus}, \quad \forall t,i,l \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus}=1 \quad (\gamma_{(t,l,i)_1}^{BA^-}) \\
v_{t,l,i}^{BA^-} - \pi_{t,l}^{A^-} \ge 0, \quad \forall t,i,l \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus}=1 \quad (\gamma_{(t,l,i)_2}^{BA^-}) \\
v_{t,l,i}^{BA^-} - \pi_{t,l}^{A^-} \le \overline{\pi}_{t,l}^{A}(1-\delta_i^{Bus}), \quad \forall t,i,l \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus}=1 \quad (\gamma_{(t,l,i)_3}^{BA^-})
\end{array}}$$

$$\boxed{\begin{array}{l}
v_{t,l,s}^{SA^-} \ge -\overline{\pi}_{t,l}^{A}\delta_s^{Sub}, \quad \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_1}^{SA^-}) \\
v_{t,l,s}^{SA^-} - \pi_{t,l}^{A^-} \ge 0, \quad \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_2}^{SA^-}) \\
v_{t,l,s}^{SA^-} - \pi_{t,l}^{A^-} \le \overline{\pi}_{t,l}^{A}(1-\delta_s^{Sub}), \quad \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_3}^{SA^-})
\end{array}}$$

$$
\begin{aligned}
&v_{t,l,ll}^{LA^-} \geq -\overline{\pi}_{t,l}^{A}\delta_{ll}^{Line}, \quad \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{Line} = 1 \quad (\gamma_{(t,l,ll)_1}^{LA^-})\\
&v_{t,l,ll}^{LA^-} - \pi_{t,l}^{A} \geq 0, \quad \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{Line} = 1 \quad (\gamma_{(t,l,ll)_2}^{LA^-})\\
&v_{t,l,ll}^{LA^-} - \pi_{t,l}^{A} \leq \overline{\pi}_{t,l}^{A}(1-\delta_{ll}^{Line}), \quad \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{Line} = 1 \quad (\gamma_{(t,l,ll)_3}^{LA^-})
\end{aligned}
$$

$$
\begin{aligned}
&v_{t,l}^{L^-} \geq -\overline{\pi}_{t,l}^{L}\delta_{l}^{Line}, \quad \forall t,l \mid l \in L^*, \mathrm{t} \in \mathrm{T}, \beta_{\mathrm{t,l}}^{\mathrm{Line}} = 1 \quad (\gamma_{(t,l)_1}^{L^-})\\
&v_{t,l}^{L^-} - \pi_{t,l}^{L^-} \geq 0, \quad \forall t,l \mid l \in L^*, \mathrm{t} \in \mathrm{T}, \beta_{\mathrm{t,l}}^{\mathrm{Line}} = 1 \quad (\gamma_{(t,l)_2}^{L^-})\\
&v_{t,l}^{L^-} - \pi_{t,l}^{L^-} \leq \overline{\pi}_{t,l}^{L}(1-\delta_{l}^{Line}), \quad \forall t,l \mid l \in L^*, \mathrm{t} \in \mathrm{T}, \beta_{\mathrm{t,l}}^{\mathrm{Line}} = 1 \quad (\gamma_{(t,l)_3}^{L^-})
\end{aligned}
$$

$$
\begin{aligned}
&v_{t,l,i}^{LB^-} \geq -\overline{\pi}_{t,l}^{L}\delta_{i}^{Bus}, \quad \forall t,i,l \mid i \in I^*, l \in L_i^{Bus}, \mathrm{t} \in \mathrm{T}, \beta_{\mathrm{t,i}}^{\mathrm{Bus}} = 1 \quad (\gamma_{(t,l,i)_1}^{LB^-})\\
&v_{t,l,i}^{LB^-} - \pi_{t,l,i}^{LB^-} \geq 0, \quad \forall t,i,l \mid i \in I^*, l \in L_i^{Bus}, \mathrm{t} \in \mathrm{T}, \beta_{\mathrm{t,i}}^{\mathrm{Bus}} = 1 \quad (\gamma_{(t,l,i)_2}^{LB^-})\\
&v_{t,l,i}^{LB^-} - \pi_{t,l,i}^{LB^-} \leq \overline{\pi}_{t,l}^{L}(1-\delta_{i}^{Bus}), \quad \forall t,i,l \mid i \in I^*, l \in L_i^{Bus}, \mathrm{t} \in \mathrm{T}, \beta_{\mathrm{t,i}}^{\mathrm{Bus}} = 1, \quad (\gamma_{(t,l,i)_3}^{LB^-})
\end{aligned}
$$

$$
\begin{aligned}
&v_{t,l,s}^{LS^-} \geq -\overline{\pi}_{t,l}^{L}\delta_{s}^{Sub}, \quad \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_1}^{LS^-})\\
&v_{t,l,s}^{LS^-} - \pi_{t,l,s}^{LS^-} \geq 0, \quad \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_2}^{LS^-})\\
&v_{t,l,s}^{LS^-} - \pi_{t,l,s}^{LS^-} \leq \overline{\pi}_{t,l}^{L}(1-\delta_{s}^{Sub}), \quad \forall t,s,l \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\gamma_{(t,l,s)_3}^{LS^-})
\end{aligned}
$$

$$
\begin{aligned}
&v_{t,l,ll}^{LL^-} \geq -\overline{\pi}_{t,l}^{L}\delta_{ll}^{Line}, \quad \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{Line} = 1 \quad (\gamma_{(t,l,ll)_1}^{LL^-})\\
&v_{t,l,ll}^{LL^-} - \pi_{t,l,ll}^{LL^-} \geq 0, \quad \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{Line} = 1 \quad (\gamma_{(t,l,ll)_2}^{LL^-})\\
&v_{t,l,ll}^{LL^-} - \pi_{t,l,ll}^{LL^-} \leq \overline{\pi}_{t,l}^{L}(1-\delta_{ll}), \quad \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{Line} = 1 \quad (\gamma_{(t,l,ll)_3}^{LL^-})
\end{aligned}
$$

$$
\begin{aligned}
&v_{t,g}^{G} \geq -\overline{\pi}_{t,g}^{G}\delta_{g}^{G}, \quad \forall t,g \mid g \in G^*, t \in T, \beta_{t,g}^{Gen} = 1 \quad (\gamma_{(t,g)_1}^{G})\\
&v_{t,g}^{G} - \pi_{t,g}^{G} \geq 0, \quad \forall t,g \mid g \in G^*, t \in T, \beta_{t,g}^{Gen} = 1 \quad (\gamma_{(t,g)_2}^{G})\\
&v_{t,g}^{G} - \pi_{t,g}^{G} \leq \overline{\pi}_{t,g}^{G}(1-\delta_{g}^{G}), \quad \forall t,g \mid g \in G^*, t \in T, \beta_{t,g}^{Gen} = 1 \quad (\gamma_{(t,g)_3}^{G})
\end{aligned}
$$

$$\boxed{\begin{aligned}&v_{t,g}^{GB} \geq -\overline{\pi}_{t,g}^{G}\delta_{i(g)}^{Bus}, \quad \forall t,g \mid i(g) \in I^*, t \in T, \beta_{t,i(g)}^{Bus} = 1 \quad (\gamma_{(t,g)_1}^{GB})\\ &v_{t,g}^{GB} - \pi_{t,g}^{GB} \geq 0, \quad \forall t,g \mid i(g) \in I^*, t \in T, \beta_{t,i(g)}^{Bus} = 1 \quad (\gamma_{(t,g)_2}^{GB})\\ &v_{t,g}^{GB} - \pi_{t,g}^{GB} \leq \overline{\pi}_{t,g}^{G}(1-\delta_{i(g)}^{Bus}), \quad \forall t,g \mid i(g) \in I^*, t \in T, \beta_{t,i(g)}^{Bus} = 1 \quad (\gamma_{(t,g)_3}^{GB})\end{aligned}}$$

$$\boxed{\begin{aligned}&v_{t,g}^{GS} \geq -\overline{\pi}_{t,g}^{G}\delta_{s(i(g))}^{Sub}, \quad \forall t,g \mid s(i(g)) \in S^*, t \in T, \beta_{t,s(i(g))}^{Sub} = 1 \quad (\gamma_{(t,g)_1}^{GS})\\ &v_{t,g}^{GS} - \pi_{t,g}^{GS} \geq 0, \quad \forall t,g \mid s(i(g)) \in S^*, t \in T, \beta_{t,s(i(g))}^{Sub} = 1 \quad (\gamma_{(t,g)_2}^{GS})\\ &v_{t,g}^{GS} - \pi_{t,g}^{GS} \leq \overline{\pi}_{t,g}^{G}(1-\delta_{s(i(g))}^{Sub}), \quad \forall t,g \mid s(i(g)) \in S^*, t \in T, \beta_{t,s(i(g))}^{Sub} = 1 \quad (\gamma_{(t,g)_3}^{GS})\end{aligned}}$$

$$\boxed{\begin{aligned}&\pi_{t,l}^{A^+} \leq \overline{\pi}_{t,l}^{A}, \quad \forall t,l \mid l \in L^*, t \in T \quad (\eta_{t,l}^{A^+})\\ &\pi_{t,l}^{L^+} \leq \overline{\pi}_{t,l}^{L}, \quad \forall t,l \mid l \in L^*, t \in T, \beta_{t,l}^{Line} = 1 \quad (\eta_{t,l}^{L^+})\end{aligned}}$$

$$\boxed{\begin{aligned}&\pi_{t,l,i}^{LB^+} \leq \overline{\pi}_{t,l}^{L}, \quad \forall t,l,i \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus} = 1 \quad (\eta_{t,l,i}^{LB^+})\\ &\pi_{t,l,s}^{LS^+} \leq \overline{\pi}_{t,l}^{L}, \quad \forall t,l,s \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\eta_{t,l,s}^{LS^+})\\ &\pi_{t,l,ll}^{LL^+} \leq \overline{\pi}_{t,l}^{L}, \quad \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{line} = 1 \quad (\eta_{t,l,ll}^{LL^+})\end{aligned}}$$

$$\boxed{\begin{aligned}&\pi_{t,l}^{A^-} \geq -\overline{\pi}_{t,l}^{A}, \quad \forall t,l \mid l \in L^*, t \in T \quad (\eta_{t,l}^{A^-})\\ &\pi_{t,l}^{L^-} \geq -\overline{\pi}_{t,l}^{L}, \quad \forall t,l \mid l \in L^*, t \in T, \beta_{t,l}^{Line} = 1 \quad (\eta_{t,l}^{L^-})\end{aligned}}$$

$$\boxed{\begin{aligned}&\pi_{t,l,i}^{LB^-} \geq -\overline{\pi}_{t,l}^{L}, \quad \forall t,l,i \mid i \in I^*, l \in L_i^{Bus}, t \in T, \beta_{t,i}^{Bus} = 1 \quad (\eta_{t,l,i}^{LB^-})\\ &\pi_{t,l,s}^{LS^-} \geq -\overline{\pi}_{t,l}^{L}, \quad \forall t,l,s \mid s \in S^*, l \in L_s^{Sub}, t \in T, \beta_{t,s}^{Sub} = 1 \quad (\eta_{t,l,s}^{LS^-})\\ &\pi_{t,l,ll}^{LL^-} \geq -\overline{\pi}_{t,l}^{L}, \quad \forall t,l,ll \mid ll \in L^*, ll \in L_l^{Par}, t \in T, \beta_{t,ll}^{line} = 1 \quad (\eta_{t,l,ll}^{LL^-})\end{aligned}}$$

$$\boxed{\begin{aligned}&\pi_{t,g}^{G} \geq -\overline{\pi}_{t,g}^{G}, \quad \forall t,g \mid g \in G^*, t \in T, \beta_{t,g}^{Gen} = 1 \quad (\eta_{t,g}^{G})\\ &\pi_{t,g}^{GB} \geq -\overline{\pi}_{t,g}^{G}, \quad \forall t,g \mid i(g) \in I^*, t \in T, \beta_{t,i(g)}^{Gen} = 1 \quad (\eta_{t,g}^{GB})\\ &\pi_{t,g}^{GS} \geq -\overline{\pi}_{t,g}^{G}, \quad \forall t,g \mid s(i(g)) \in S^*, t \in T, \beta_{t,s(i(g))}^{Gen} = 1 \quad (\eta_{t,g}^{GS})\end{aligned}}$$

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